COMPETENCY 1
KNOWLEDGE OF NUMBER SENSE, CONCEPTS, AND OPERATIONS

SKILL 1.1 Compare the relative value of real numbers *(e.g., integers, fractions, decimals, percents, irrational numbers, and numbers expressed in exponential or scientific notation)*

Rational numbers can be expressed as the ratio of two integers, $\frac{a}{b}$, where $b \neq 0$, for example, $\frac{2}{3}, -\frac{4}{5}, 5 = \frac{5}{1}$ are all rational numbers.

Rational numbers include integers, fractions, mixed numbers, and terminating and repeating decimals. Every rational number can be expressed as a repeating or terminating decimal and can be shown on a number line.

INTEGERS are the positive and negative whole numbers and zero.

...−6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6,...

WHOLE NUMBERS are the natural numbers and zero.

0, 1, 2, 3, 4, 5, 6...

NATURAL NUMBERS are the counting numbers.

1, 2, 3, 4, 5, 6...

IRRATIONAL NUMBERS are real numbers that cannot be written as the ratio of two integers. They are infinite, nonrepeating decimals.

$\sqrt{5} = 2.2360$, pi $= \pi = 3.1415927...$

A **FRACTION** is an expression of numbers in the form of $\frac{x}{y}$, where x is the numerator and y is the denominator, which cannot be zero.

$\frac{3}{7}$ 3 is the numerator; 7 is the denominator

If the fraction has common factors for the numerator and denominator, divide both by the common factor to reduce the fraction to its lowest form.

$\frac{13}{39} = \frac{1 \times 13}{3 \times 13} = \frac{1}{3}$ Divide by the common factor 13.

A **MIXED NUMBER** has an integer part and a fractional part.

$2\frac{1}{4}, -5\frac{1}{6}, 7\frac{1}{3}$

PERCENT means per 100 (written with the symbol %).

$10\% = \frac{10}{100} = \frac{1}{10}$

INTEGERS: the positive and negative whole numbers and zero

WHOLE NUMBERS: the natural numbers and zero

NATURAL NUMBERS: the counting numbers

IRRATIONAL NUMBERS: real numbers that cannot be written as the ratio of two integers

FRACTION: an expression of numbers in the form of $\frac{x}{y}$, where x is the numerator and y is the denominator

MIXED NUMBER: a number that has an integer part and a fractional part

PERCENT: means “per 100;” ten percent is 10 parts out of 100

DECIMAL: a number written with a whole-number part, a decimal point, and a decimal part

EXPONENT FORM: a shorthand way of writing repeated multiplication

BASE: the number to be multiplied as many times as indicated by the exponent

EXPONET: tells how many times the base is multiplied by itself

DECIMALS means deci or part of ten. To find the decimal equivalent of a fraction, use the denominator to divide the numerator, as shown in the following example.

Find the decimal equivalent of $\frac{7}{10}$.

Because 10 cannot divide into 7 evenly, $\frac{7}{10} = 0.7$

The **EXPONENT FORM** is a shortcut method to write repeated multiplication. The basic form is b^n, where b is called the **BASE** and n is the **EXPONENT**. Both b and n are real numbers. The b^n implies that the base b is multiplied by itself n times.

Examples:

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$2^3 = 2 \times 2 \times 2 = 8$

$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$

$-2^4 = -(2 \times 2 \times 2 \times 2) = 16$

Caution: The exponent does not affect the sign unless the negative sign is inside the parentheses and the exponent is outside the parentheses.

$(-2)^4$ implies that -2 is multiplied by itself 4 times.

-2^4 implies that 2 is multiplied by itself 4 times, and then the answer becomes. negative.

KEY EXPONENT RULES: FOR *'a'* NONZERO AND *'m'* AND *'n'* REAL NUMBERS	
Product Rule	$a^m \times a^n = a^{(m+n)}$
Quotient Rule	$\frac{a^m}{a^n} = a^{(m-n)}$
Rule of Negative Exponents	$\frac{a^{-m}}{a^{-n}} = \frac{a^n}{a^m}$

When 10 is raised to any power, the exponent tells the numbers of zeros in the product.

Example:

$10^7 = 10{,}000{,}000$

SCIENTIFIC NOTATION: a convenient method for writing very large and very small numbers

SCIENTIFIC NOTATION is a convenient method for writing very large and very small numbers. It employs two factors. The first factor is a number between 1 and 10. The second factor is a power of 10. This notation is considered "shorthand" for expressing very large numbers (such as the weight of 100 elephants) or very small numbers (such as the weight of an atom in pounds).

Recall that:

10^n	=	Ten multiplied by itself n times
10^n	=	Any nonzero number raised to the zero power is 1
10^1	=	10
10^2	=	$10 \times 10 = 100$
10^3	=	$10 \times 10 \times 10 = 1000$
10^{-1}	=	$\frac{1}{10}$ (deci)
10^{-2}	=	$\frac{1}{100}$ (centi)
10^{-3}	=	$\frac{1}{1000}$ (milli)
10^{-6}	=	$\frac{1}{1{,}000{,}000}$ (micro)

Example: Write 46,368,000 in scientific notation.

1. Introduce a decimal point and decimal places.
   46,368,000 = 46,368,000.0000

2. Make a mark between the two digits that give a number between −9.9 and 9.9.
   4 ∧ 6,368,000.0000

3. Count the number of digit places between the decimal point and the ∧ mark. This number is the *nth* power of ten.
   So, $46{,}368{,}000 = 4.6368 \times 10^7$.

Example: Write 0.00397 in scientific notation.

1. Decimal place is already in place.

2. Make a mark between 3 and 9 to obtain a number between −9.9 and. 9.9.

3. Move decimal place to the mark (three hops).
   0.003 ∧ 97
   Motion is to the right, so n on 10^n is negative.
   Therefore, $0.00397 = 3.97 \times 10^{-3}$.

SKILL 1.2 Solve real-world problems involving addition, subtraction, multiplication, and division of rational numbers *(e.g., whole numbers, integers, decimals, percents, and fractions including mixed numbers)*

PROPERTIES: rules that apply for addition, subtraction, multiplication, or division of real numbers

PROPERTIES are rules that apply for addition, subtraction, multiplication, or division of real numbers. These properties are:

Commutative	You can change the order of the terms or factors as follows. **For addition:** $a + b = b + a$ **For multiplication:** $ab = ba$ Since addition is the inverse operation of subtraction and multiplication is the inverse operation of division, no separate laws are needed for subtraction and division. *Example:* $5 + 8 = 8 + 5 = 13$ *Example:* $2 \times 6 = 6 \times 2 = 12$
Associative	You can regroup the terms as you like. **For addition:** $a + (b + c) = (a + b) + c$ **For multiplication:** $a(bc) = (ab)c$ This rule does not apply for division and subtraction. *Example:* $(2 + 7) + 5 = 2 + (7 + 5)$ $9 + 5 = 2 + 12 = 14$ *Example:* $(3 \times 7) \times 5 = 3 \times (7 \times 5)$ $21 \times 5 = 3 \times 35 = 105$
Identity	Finding a number so that when added to a term results in that number (additive identity); finding a number such that when multiplied by a term results in that number (multiplicative identity). **For addition:** $a + 0 = a$ (zero is additive identity) **For multiplication:** $a \times 1 = a$ (one is multiplicative) *Example:* $17 + 0 = 17$ *Example:* $34 \times 1 = 34$ The product of any number and one is that number.

Continued on next page

Inverse	Finding a number such that when added to the number, it results in zero; or when multiplied by the number, it results in 1. **For addition:** $a - a = 0$ **For multiplication:** $a \times (\frac{1}{a}) = 1$ ($-a$) is the additive inverse of a; ($\frac{1}{a}$), also called the reciprocal, is the multiplicative inverse of a. *Example:* $25 - 25 = 0$ *Example:* $5 \times \frac{1}{5} = 1$ The product of any number and its reciprocal is one.
Distributive	This technique allows us to operate on terms within parentheses without first performing operations within the parentheses. This is especially helpful when terms within the parentheses cannot be combined. $a(b + c) = ab + ac$ *Example:* $6 \times (4 + 9) = (6 \times 4) + (6 \times 9)$ $6 \times 13 = 24 + 54 = 78$ To multiply a sum by a number, multiply each addend by the number, then add the products.

Addition of Whole Numbers

Example: At the end of a day of shopping, a shopper had $24 remaining in his wallet. He spent $45 on various goods. How much money did the shopper have at the beginning of the day?

The total amount of money the shopper started with is the sum of the amount spent and the amount remaining at the end of the day.

$$\begin{array}{r} \$\ 24 \\ +\ 45 \\ \hline \$\ 69 \end{array}$$

The original total was $69.

Example: A race took the winner 1 hr. 58 min. 12 sec. on the first half of the race and 2 hr. 9 min. 57 sec. on the second. How much time did the entire race take?

1 hr 58 min 12 sec	
+ 2 hr 9 min 57 sec	Add these numbers.
3 hr 67 min 69 sec	
+ 1 min − 60 sec	Change 60 sec to 1 min.
3 hr 68 min 9 sec	
+ 1 hr − 60 min	Change 60 min to 1 hr.
4 hr 8 min 9 sec	Final answer.

Subtraction of Whole Numbers

Example: At the end of his shift, a cashier has $96 in the cash register. At the beginning of his shift, he had $15. How much money did the cashier collect during his shift?

The total collected is the difference between the ending amount and the starting amount.

$ 96	
−15	
$ 81	The total collected was $81.

Multiplication of Whole Numbers

Another way of conceptualizing multiplication is to think in terms of groups.

PRODUCT: the answer to a multiplication problem

Multiplication is one of the four basic number operations. In simple terms, multiplication is the addition of a number to itself a certain number of times. For example, 4 multiplied by 3 is equal to 4 + 4 + 4 or 3 + 3 + 3 + 3. Another way of conceptualizing multiplication is to think in terms of groups. For example, if we have 4 groups of 3 students, the total number of students is 4 multiplied by 3. We call the solution to a multiplication problem the **PRODUCT**.

The basic algorithm for whole number multiplication begins with aligning the numbers by place value, with the number containing more places on top.

172	
× 43	Note that we placed 172 on top because it has more places than 43 does.

Next, we multiply the ones place of the bottom number by each place value of the top number sequentially.

(2)	
172	{3 × 2 = 6, 3 × 7 = 21, 3 × 1 = 3}
× 43	Note that we had to carry a 2 to the hundreds column
516	because 3 × 7 = 21. Note also that we add carried numbers to the product.

Next, we multiply the number in the tens place of the bottom number by each place value of the top number sequentially. Because we are multiplying by a number in the tens place, we place a zero at the end of this product.

(2)
172
$\underline{\times 43}$ $\{4 \times 2 = 8, 4 \times 7 = 28, 4 = 1 = 4\}$
516
6880

Finally, to determine the final product, we add the two partial products.

172
$\underline{\times 43}$
516
$\underline{+ 6880}$
7396 The product of 172 and 43 is 7,396.

Example: A student buys 4 boxes of crayons. Each box contains 16 crayons. How many total crayons does the student have?

The total number of crayons is 16×4.

16
$\underline{\times 4}$
64 The total number of crayons equals 64.

Division of Whole Numbers

Division, the inverse of multiplication, is another of the four basic number operations. When we divide one number by another, we determine how many times we can multiply the divisor (number divided by) before we exceed the number we are dividing (dividend). For example, 8 divided by 2 equals 4 because we can multiply 2 four times to reach 8 ($2 \times 4 = 8$ or $2 + 2 + 2 + 2 = 8$). Using the grouping conceptualization we used with multiplication, we can divide 8 into 4 groups of 2 or 2 groups of 4. We call the answer to a division problem the **QUOTIENT**.

QUOTIENT: the answer to a division problem

If the divisor does not divide evenly into the dividend, we express the leftover amount either as a remainder or as a fraction with the divisor as the denominator. For example, 9 divided by 2 equals 4 with a remainder of 1, or $4\frac{1}{2}$.

The basic algorithm for division is long division. We start by representing the quotient as follows.

$14\overline{)293} \rightarrow$ 14 is the divisor and 293 is the dividend.
This represents $293 \div 14$.

Next, we divide the divisor into the dividend, starting from the left.

$$\begin{array}{r} 2 \\ 14\overline{)293} \end{array} \rightarrow \text{ 14 divides into 29 two times with a remainder.}$$

Next, we multiply the partial quotient by the divisor, subtract this value from the first digits of the dividend, and bring down the remaining dividend digits to complete the number.

$$\begin{array}{r} 2 \\ 14\overline{)293} \\ \underline{-\,28}\downarrow \\ 13 \end{array} \rightarrow 2 \times 14 = 28,\ 29 - 28 = 1, \text{ and bringing down the 3 yields 13.}$$

Finally, we divide again (the divisor into the remaining value) and repeat the preceding process. The number left after the subtraction represents the remainder.

$$\begin{array}{r} 20 \\ 14\overline{)293} \\ \underline{-\,28} \\ 13 \\ \underline{-0} \\ 13 \end{array} \rightarrow$$

The final quotient is 20 with a remainder of 13. We can also represent this quotient as $20\,\frac{13}{14}$.

Example: Each box of apples contains 24 apples. How many boxes must a grocer purchase to supply a group of 252 people with one apple each?

The grocer needs 252 apples. Because he must buy apples in groups of 24, we divide 252 by 24 to determine how many boxes he needs to buy.

$$\begin{array}{r} 10 \\ 24\overline{)252} \\ \underline{-\,24} \\ 12 \\ \underline{-0} \\ 12 \end{array}$$

12 → The quotient is 10 with a remainder of 12.

Thus, the grocer needs 10 boxes plus 12 more apples. Therefore, the minimum number of boxes the grocer can purchase is 11.

Example: At his job, John gets paid $20 for every hour he works. If John made $940 in a week, how many hours did he work?

This is a division problem. To determine the number of hours John worked, we divide the total amount made ($940) by the hourly rate of pay ($20). Thus, the number of hours worked equals 940 divided by 20.

```
    47
20)940
 − 80
   140
  −140
     0 →  20
```
20 Divides into 940 a total of 47 times with no remainder.

John worked 47 hours.

Addition and Subtraction of Decimals

When adding and subtracting decimals, we align the numbers by place value as we do with whole numbers. After adding or subtracting each column, we bring the decimal down, placing it in the same location as in the numbers added or subtracted.

When adding and subtracting decimals, we align the numbers by place value as we do with whole numbers.

Example: Find the sum of 152.3 and 36.342.

```
  152.300
+ 36.342
  188.642
```

Note that we placed two zeros after the final place value in 152.3 to clarify the column addition.

Example: Find the difference of 152.3 and 36.342.

```
      2 9 10        (4)11(12)
  152.~~300~~       1~~52.300~~
− 36.342          − 36.342
       58           115.958
```

Note how we borrowed to subtract from the zeros in the hundredths and thousandths places of 152.300.

Multiplication of Decimals

When multiplying decimal numbers, we multiply exactly as with whole numbers and place the decimal in from the right the total number of decimal places contained in the two numbers multiplied. For example, when multiplying 1.5 and 2.35, we place the decimal in the product 3 places in from the right (3.525).

Example: Find the product of 3.52 and 4.1.

$$\begin{array}{r} 3.52 \\ \times\ 4.1 \\ \hline 352 \\ +\ 14080 \\ \hline 14.432 \end{array}$$

Note that there are three decimal places in total in the two numbers.

We place the decimal three places in from the right.

Thus, the final product is 14.432.

Example: A shopper has 5 one-dollar bills, 6 quarters, 3 nickels, and 4 pennies in his pocket. How much money does he have?

$5 \times \$1.00 = \$5.00$

$$\begin{array}{rrr} {}^{1\,3} & {}^{1} & \\ \$0.25 & \$0.05 & \$0.01 \\ \times\ 6 & \times\ 3 & \times\ 4 \\ \hline \$1.50 & \$0.15 & \$0.04 \end{array}$$

Note the placement of the decimals in the multiplication products. Thus, the total amount of money in the shopper's pocket is:

$$\begin{array}{r} \$5.00 \\ 1.50 \\ 0.15 \\ +\ 0.04 \\ \hline \$6.69 \end{array}$$

Division of Decimals

When dividing decimal numbers, we first remove the decimal in the divisor by moving the decimal in the dividend the same number of spaces to the right. For example, when dividing 1.45 into 5.3, we convert the numbers to 145 and 530 and perform normal whole-number division.

Example: Find the quotient of 5.3 divided by 1.45.

Convert to 145 and 530.

Divide.

$$\begin{array}{r} 3 \\ 145\overline{)530} \\ -435 \\ \hline 95 \end{array} \qquad \begin{array}{r} 3.65 \\ 145\overline{)530.00} \\ -435 \\ \hline 950 \\ -870 \\ \hline 800 \end{array}$$

Note that we insert the decimal to continue division.

Because one of the numbers divided contained one decimal place, we round the quotient to one decimal place. Thus, the final quotient is 3.7.

Operating with Percents

Example: 5 is what percent of 20?

This is the same as converting $\frac{5}{20}$ to % form.

$\frac{5}{20} \times \frac{100}{1} = \frac{5}{1} \times \frac{5}{1} = 25\%$

Example: There are 64 dogs in the kennel. 48 are collies. What percent are collies?

Restate the problem.	48 is what percent of 64?
Write an equation.	$48 = n \times 64$
Solve.	$\frac{48}{64} = n$

$n = \frac{3}{4} = 75\%$

75% of the dogs are collies.

Example: The auditorium was filled to 90% capacity. There were 558 seats occupied. What is the capacity of the auditorium?

Restate the problem.	90% of what number is 558?
Write an equation.	$0.9n = 558$
Solve.	$n = \frac{558}{.9}$

$n = 620$

The capacity of the auditorium is 620 people.

Example: A pair of shoes costs $42.00. The sales tax is 6%. What is the total cost of the shoes?

Restate the problem.	What is 6% of 42?
Write an equation.	$n = 0.06 \times 42$
Solve.	$n = 2.52$
Add the sales tax to the cost.	$42.00 + $2.52 = $44.52

The total cost of the shoes, including sales tax, is $44.52.

Addition and Subtraction of Fractions

Key points

1. You need a common denominator in order to add and subtract reduced and improper fractions.

   Example:

   $\frac{1}{3} + \frac{7}{3} = \frac{1+7}{3} = \frac{8}{3} = 2\frac{2}{3}$

   Example:

   $\frac{4}{12} + \frac{6}{12} - \frac{3}{12} = \frac{4+6-3}{12} = \frac{7}{12}$

2. Adding an integer and a fraction of the same sign results directly in a mixed fraction.

   Example:

   $2 + \frac{2}{3} = 2\frac{2}{3}$

   Example:

   $-2 - \frac{2}{3} = -2\frac{2}{3}$

3. Adding an integer and a fraction with different signs involves the following steps.

   - Get a common denominator
   - Add or subtract as needed
   - Change to a mixed fraction if possible

   Example:

   $2 - \frac{1}{3} = \frac{2 \times 3 - 1}{3} = \frac{6 - 1}{3} = \frac{5}{3} = 1\frac{2}{3}$

   Example:

   Add $7\frac{3}{8} + 5\frac{2}{7}$

   Add the whole numbers, add the fractions, and combine the two results:

   $7\frac{3}{8} + 5\frac{2}{7} = (7 + 5) + (\frac{3}{8} + \frac{2}{7})$

   $= 12 + \frac{(7 \times 3) + (8 \times 2)}{56}$ (LCM of 8 and 7)

   $= 12 + \frac{21 + 16}{56} = 12 + \frac{37}{56} = 12\frac{37}{56}$

   Example: Perform the operation.

   $\frac{2}{3} - \frac{5}{6}$

   We first find the LCM of 3 and 6, which is 6.

   $\frac{2 \times 2}{3 \times 2} - \frac{5}{6} \rightarrow \frac{4 - 5}{6} = \frac{-1}{6}$ (Using method A)

   Example:

   $-7\frac{1}{4} + 2\frac{7}{8}$

   $-7\frac{1}{4} + 2\frac{7}{8} = (-7 + 2) + (\frac{-1}{4} + \frac{7}{8})$

   $= (-5) + \frac{-2 + 7}{8} = (-5) + (\frac{5}{8})$

   $= (-5) + \frac{5}{8} = \frac{-5 \times 8}{1 \times 8} + \frac{5}{8} = \frac{-40 + 5}{8}$

   $= \frac{-35}{8} = -4\frac{3}{8}$

   Divide 35 by 8 to get 4, remainder 3.