## PART I. UNDERSTANDING AND SKILL IN PHYSICS

## SUBAREA I. MOTION AND FORCES

### COMPETENCY 1.1 MOTION AND FORCES

**Skill 1.1a Discuss and apply Newton's laws (i.e., first, second, third, and law of universal gravitation)**

**Newton's first law of motion:** "An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force". This tendency of an object to continue in its state of rest or motion is known as **inertia**. Note that, at any point in time, most objects have multiple forces acting on them. If the vector addition of all the forces on an object results in a zero net force, then the forces on the object are said to be **balanced**. If the net force on an object is non-zero, an **unbalanced** force is acting on the object.

Prior to Newton's formulation of this law, being at rest was considered the natural state of all objects because at the earth's surface we have the force of gravity working at all times which causes nearly any object put into motion to eventually come to rest. Newton's brilliant leap was to recognize that an unbalanced force changes the motion of a body, whether that body begins at rest or at some non-zero speed.

We experience the consequences of this law everyday. For instance, the first law is why seat belts are necessary to prevent injuries. When a car stops suddenly, say by hitting a road barrier, the driver continues on forward due to inertia until acted upon by a force. The seat belt provides that force and distributes the load across the whole body rather than allowing the driver to fly forward and experience the force against the steering wheel.

**Newton's second law of motion**: "The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object". In the equation form, it is stated as $F = ma$, force equals mass times acceleration. It is important, again, to remember that this is the net force and that forces are vector quantities. Thus if an object is acted upon by 12 forces that sum to zero, there is no acceleration. Also, this law embodies the idea of inertia as a consequence of mass. For a given force, the resulting acceleration is proportionally smaller for a more massive object because the larger object has more inertia.

The first two laws are generally applied together via the equation **F=ma.** The first law is largely the conceptual foundation for the more specific and quantitative second law.

The **weight** of an object is the result of the gravitational force of the earth acting on its mass. The acceleration due to Earth's gravity on an object is 9.81 $m/s^2$. Since force equals mass * acceleration, the magnitude of the gravitational force created by the earth on an object is

$$F_{Gravity} = m_{object} \cdot 9.81 \, m/s^2$$

**Newton's third law of motion**: **"**For every action, there is an equal and opposite reaction". This statement means that, in every interaction, there is a pair of forces acting on the two interacting objects. The size of the force on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object.

**1. The propulsion/movement of fish through water:** A fish uses its fins to push water backwards. The water pushes back on the fish. Because the force on the fish is unbalanced the fish moves forward.
**2. The motion of car:** A car's wheels push against the road and the road pushes back. Since the force of the road on the car is unbalanced the car moves forward.
**3. Walking:** When one pushes backwards on the foot with the muscles of the leg, the floor pushes back on the foot. If the forces of the leg on the foot and the floor on the foot are balanced, the foot will not move and the muscles of the body can move the other leg forward.

**Newton's universal law of gravitation** states that any two objects experience a force between them as the result of their masses. Specifically, the force between two masses $m_1$ and $m_2$ can be summarized as

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the gravitational constant ($G = 6.672 \times 10^{-11} Nm^2 / kg^2$), and r is the distance between the two objects.

Important things to remember:

1. The gravitational force is proportional to the masses of the two objects, but *inversely* proportional to the *square of the distance* between the two objects.
2. When calculating the effects of the acceleration due to gravity for an object above the earth's surface, the distance above the surface is ignored because it is inconsequential compared to the radius of the earth. The constant figure of 9.81 $m/s^2$ is used instead.

Problem: Two identical 4 kg balls are floating in space, 2 meters apart. What is the magnitude of the gravitational force they exert on each other?

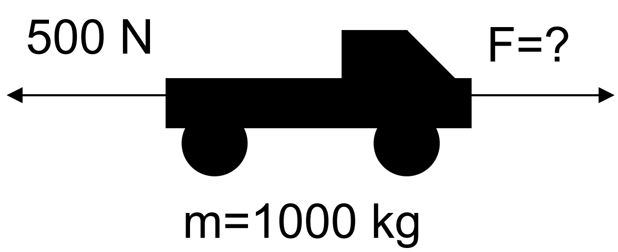
Solution:

$$F = G\frac{m_1 m_2}{r^2} = G\frac{4\times 4}{2^2} = 4G = 2.67\times 10^{-10}\,N$$

**Skill 1.1b Solve problems using Newton's Second Law (e.g., problems involving time, velocity, and space-dependent forces)**

**Problems involving constant force**

1. For the arrangement shown, find the force necessary to overcome the 500 N force pushing to the left and move the truck to the right with an acceleration of 5 m/s$^2$.




Since we know the acceleration and mass, we can calculate the net force necessary to move the truck with this acceleration. Assuming that to the right is the positive direction we sum the forces and get
F-500N = 1000kg x 5 m/s$^2$. Solving for F, we get 5500 N.

2. A 30 kg woman is in a car accident. She was driving at 50m/s when she had to hit the brakes to avoid hitting the car in front of her.

a. The automatic tensioning device in her seatbelt slows her down to a stop over a period of one half second. How much force does it apply?

$$F = m\cdot\frac{\Delta v}{t} \rightarrow F = 30\cdot\frac{50}{.5} = 3000N$$

b. If she hadn't been wearing a seatbelt, the windshield would have stopped her in .001 seconds. How much force would have been applied there?

$$F = m\cdot\frac{\Delta v}{t} \rightarrow F = 30\cdot\frac{50}{.001} = 1500000N$$

**Problems involving varying force**

1. A falling skydiver of mass 75 Kg experiences a drag force due to air resistance that is proportional to the square of his velocity and is given by $f_{drag} = -\frac{1}{2}C\rho Av^2$ where the drag coefficient C=0.5 and the density of air is given by $\rho = 1.29 Kg/m^3$. If the cross-sectional area of the skydiver is approximately 0.7 sq.m, what is his terminal velocity?

   As the skydiver falls, his velocity increases and so does the drag force which is proportional to the square of the velocity. When the drag force reaches a value equal to the weight of the skydiver, the net force on him is zero and the velocity remains at a constant value known as the terminal velocity. Thus, if $v_t$ is the terminal velocity, the net force on the skydiver at that point is given by

   $$mg - \frac{1}{2}C\rho A{v_t}^2 = 0$$

   Thus, $v_t = \sqrt{\frac{2mg}{C\rho A}} = \sqrt{\frac{2\times 75\times 9.8}{0.5\times 1.29\times 0.7}} = 57m/s$

2. The restoring force on an object attached to a spring is given by Hooke's law $F = -kx$ and is dependent on the displacement of the object. Since the acceleration of the mass is given by $a = \frac{d^2x}{dt^2}$, using Newton's second law of motion we get $ma = -kx$ or $m\frac{d^2x}{dt^2} + kx = 0$.

   For a solution to the motion of the mass-spring system see **Skill 1.1k.**

**Skill 1.1c Define pressure and relate it to fluid flow and buoyancy (e.g., heart valves, atmospheric pressure)**

The weight of a column of fluid creates hydrostatic pressure. Common situations in which we might analyze hydrostatic pressure include tanks of fluid, a swimming pool, or the ocean. Also, **atmospheric pressure** is an example of hydrostatic pressure. Because hydrostatic pressure results from the force of gravity interacting with the mass of the fluid or gas, for an incompressible fluid it is governed by the following equation:

$$P = \rho gh$$

where P=hydrostatic pressure
ρ=density of the fluid
g=acceleration of gravity
h=height of the fluid column

Example**:** How much pressure is exerted by the water at the bottom of a 5 meter swimming pool filled with water?

Solution**:** We simply use the equation from above, recalling that the acceleration due to gravity is 9.8m/s$^2$ and the density of water is 1000 kg/m$^3$.

$$P = \rho gh = 1000\frac{kg}{m^3} \times 9.8\frac{m}{s^2} \times 5m = 49{,}000Pa = 49kPa$$

**Archimedes' Principle** states that, for an object in a fluid, "the upthrust is equal to the weight of the displaced fluid". Today, we call Archimedes' "upthrust" **buoyancy**. Buoyancy is the force produced by a fluid on a fully or partially immersed object. The buoyant force ($F_{buoyant}$) is found using the following equation:

$$F_{bouyant} = \rho Vg$$

where $\rho$=density of the fluid
V=volume of fluid displaced by the submerged object
g=the acceleration of gravity

Notice that the buoyant force opposes the force of gravity. For an immersed object, a gravitational force (equal to the object's mass times the acceleration of gravity) pulls it downward, while a buoyant force (equal to the weight of the displaced fluid) pushes it upward.

Also note that, from this principle, we can predict *whether* an object will sink or float in a given liquid. We can simply compare the density of the material from which the object is made to that of the liquid. If the material has a lower density, it will float; if it has a higher density it will sink. Finally, if an object has a density equal to that of the liquid, it will neither sink nor float.

Example**:** Will gold ($\rho$=19.3 g/cm$^3$) float in water?

Solution**:** We must compare the density of gold with that of water, which is 1 g/cm$^3$.

$$\rho_{gold} > \rho_{water}$$

So, gold will sink in water.

Example**:** Imagine a 1 $m^3$ cube of oak (530 kg/$m^3$) floating in water. What is the buoyant force on the cube and how far up the sides of the cube will the water be?

Solution**:** Since the cube is floating, it has displaced enough water so that the buoyant force is equal to the force of gravity. Thus the buoyant force on the cube is equal to its weight 1X530X9.8 N = 5194 N.

To determine where the cube sits in the water, we simply the find the ratio of the wood's density to that of the water:

$$\frac{\rho_{oak}}{\rho_{water}} = \frac{530\,{}^{kg}\!/_{m^3}}{1000\,{}^{kg}\!/_{m^3}} = 0.53$$

Thus, 53% of the cube will be submerged. Since the edges of the cube must be 1m each, the top 0.47m of the cube will appear above the water.

Much of what we know about fluid flow today was originally discovered by Daniel Bernoulli. His most famous discovery is known as **Bernoulli's Principle** which states that, if no work is performed on a fluid or gas, an increase in velocity will be accompanied by a decrease in pressure. The mathematical statement of the Bernoulli's Principle for incompressible flow is:

$$\frac{v^2}{2} + gh + \frac{p}{\rho} = \text{constant}$$

where $v$= fluid velocity
$g$=acceleration due to gravity
$h$=height
$p$=pressure
$\rho$=fluid density

Though some physicists argue that it leads to the compromising of certain assumptions (i.e., incompressibility, no flow motivation, and a closed fluid loop), most agree it is correct to explain "lift" using Bernoulli's principle. This is because Bernoulli's principle can also be thought of as predicting that the pressure in moving fluid is less than the pressure in fluid at rest. Thus, there are many examples of physical phenomenon that can be explained by Bernoulli's Principle:

- The lift on airplane wings occurs because the top surface is curved while the bottom surface is straight. Air must therefore move at a higher velocity on the top of the wing and the resulting lower pressure on top accounts for lift.

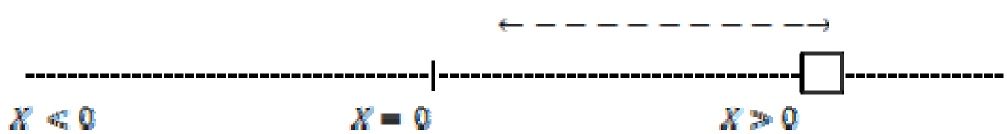
- The tendency of windows to explode rather than implode in hurricanes is caused by the pressure drop that results from the high speed winds blowing across the outer surface of the window. The higher pressure on the inside of the window then pushes the glass outward, causing an explosion.
- The ballooning and fluttering of a tarp on the top of a semi-truck moving down the highway is caused by the flow of air across the top of the truck. The decrease in pressure causes the tarp to "puff up."
- A perfume atomizer pushes a stream of air across a pool of liquid. The drop in pressure caused by the moving air lifts a bit of the perfume and allows it to be dispensed.

Flow of incompressible fluids is also governed by the **equation of continuity** $v_1 A_1 = v_2 A_2$ which arises from the conservation of mass and states that the product of the cross-sectional area of a pipe and the velocity of the fluid flowing through it must be constant. This means that the fluid will flow faster in the narrower portions of the pipe and more slowly in the wider regions.

The equation of continuity, along with Bernoulli's equation, may be used to explain blood flow characteristics in artificial heart valves. As blood flows through the narrow area in a valve, the velocity of flow increases (equation of continuity) and results in a pressure drop (Bernoulli's equation). Artificial valves are designed to minimize this effect. Other potential problems with artificial valves may also be modeled using fluid dynamics.

**Skill 1.1d Describe the relationships among position, distance, displacement, speed, velocity, acceleration, and time, and perform simple calculations using these variables for both linear and circular motion**

Kinematics is the part of mechanics that seeks to understand the motion of objects, particularly the relationship between position, velocity, acceleration and time.




The above figure represents an object and its displacement along one linear dimension.

First we will define the relevant terms:

**1. Position or Distance** is usually represented by the variable *x*. It is measured relative to some fixed point or datum called the origin in linear units, meters, for example.

***2.*** **Displacement** is defined as the change in position or distance which an object has moved and is represented by the variables D, d or $\Delta x$. Displacement is a vector with a magnitude and a direction.
***3.*** **Velocity** is a vector quantity usually denoted with a V or v and defined as the rate of change of position. Typically units are distance/time, m/s for example. Since velocity is a vector, if an object changes the direction in which it is moving it changes its velocity even if the speed (the scalar quantity that is the magnitude of the velocity vector) remains unchanged.

**i) Average velocity:** $\vec{v} = \frac{\Delta d}{\Delta t} = d_1 - d_0 / t_1 - t_0$.

The ratio, $\Delta d/\Delta t$ is called the average velocity. Average here denotes that this quantity is defined over a period $\Delta t$.
**ii) Instantaneous velocity** is the velocity of an object at a particular moment in time. Conceptually, this can be imagined as the extreme case when $\Delta t$ is infinitely small.
**5. Acceleration,** represented by *a,* is defined as the rate of change of velocity and the units are m/$s^2$. Both an average and an instantaneous acceleration can be defined similarly to velocity.

From these definitions we develop the kinematic equations. In the following, subscript i denotes initial and subscript f denotes final values for a time period. Acceleration is assumed to be constant with time.

$$v_f = v_i + at \qquad (1)$$

$$d = v_i t + \frac{1}{2}at^2 \qquad (2)$$

$$v_f^2 = v_i^2 + 2ad \qquad (3)$$

$$d = \left(\frac{v_i + v_f}{2}\right)t \qquad (4)$$

Problem:
Leaving a traffic light a man accelerates at 10 m/s$^2$. a) How fast is he going when he has gone 100 m? b) How fast is he going in 4 seconds? C) How far does he travel in 20 seconds.

Solution**:**
a) Use equation 3. He starts from a stop so $v_i$=0 and $v_f^2$=2 x 10m/s$^2$ x 100m=2000 m$^2$/s$^2$ and $v_f$=45 m/s.
b) Use equation 1. Initial velocity is again zero so $v_f$=10m/s$^2$ x 4s=40 m/s.
c) Use equation 2. Since initial velocity is again zero, d=1/2 x 10 m/s$^2$ x (20s)$^2$=2000 m

Motion on an arc can also be considered from the view point of the kinematic equations. Linear motion is measured in rectangular coordinates. Rotational motion is measured differently, in terms of the angle of displacement. There are three common ways to measure rotational displacement; degrees, revolutions, and radians. Degrees and revolutions have an easy to understand relationship, one revolution is 360°. Radians are slightly less well known and are defined as $\frac{arc\ length}{radius}$. Therefore 360°=2π radians and 1 radian = 57.3°.

The major concepts of linear motion are duplicated in rotational motion with linear displacement replaced by **angular displacement** $\theta$.

**Angular velocity** $\omega =$ rate of change of angular displacement.
**Angular acceleration** $\alpha =$ rate of change of angular velocity.

The kinematic equations for circular motion with constant angular acceleration are exactly analogous to the linear equations and are given by

$$\omega_f = \omega_i + \alpha t$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f{}^2 = \omega_i{}^2 + 2\alpha\theta$$

Problem:
A wheel is rotating at the rate of 1 revolution in 8 seconds. If a constant deceleration is applied to the wheel it stops in 7 seconds. What is the deceleration applied to the wheel?

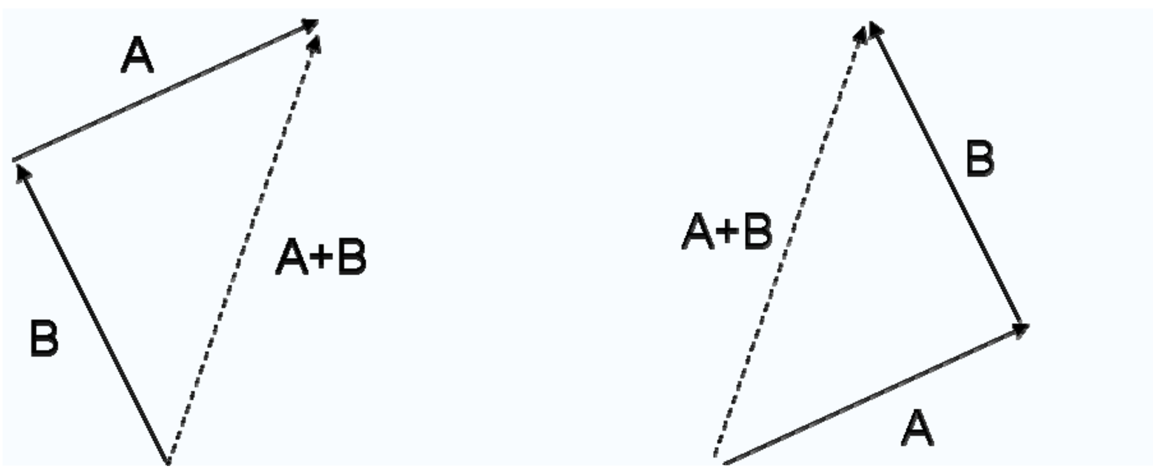
Solution:
Initial angular velocity = 2π/8 radians/sec = 0.25π radians/sec.
Final angular velocity = 0 radians/sec.
Using the rotational kinematic equations, angular acceleration applied to the wheel = (0 - 0.25x3.14)/7 = -0.11 radian/ (sec.sec)

**Skill 1.1e Construct and analyze simple vector and graphical representations of motion and forces (e.g., distance, speed, time)**

Vectors are used in representing any quantity that has both magnitude and direction. This is why both velocities and forces are expressed as vector quantities. It is not sufficient, for example, to describe a car as traveling south; we must know that it is traveling south at 50 miles per hour to fully describe its motion. Similarly, we must know not only that the force of gravity has a magnitude of 9.8 m/s$^2$ times the mass of an object, but that it is directed toward the center of the earth.

When we wish to analyze a physical situation involving vector quantities such as force and velocity, the first step is typically the creation of a diagram. The objects involved are drawn and arrows are used to represent the vector quantities, which are labeled appropriately. In precise diagrams drawn to scale on graph paper, the magnitude of each force is indicated by the length of the arrow and the exact angles between the vectors will be depicted. Otherwise, angles and magnitudes may simply be noted on the diagram.

To add two vectors graphically, the base of the second vector is drawn from the point of the first vector as shown below with vectors **A** and **B**. The sum of the vectors is drawn as a dashed line, from the base of the first vector to the tip of the second. As illustrated, the order in which the vectors are connected is not significant as the endpoint is the same graphically whether **A** connects to **B** or **B** connects to **A**. This principle is sometimes called the parallelogram rule.




If more than two vectors are to be combined, additional vectors are simply drawn in accordingly with the sum vector connecting the base of the first to the tip of the final vector.