SUBAREA I. FOUNDATIONS OF MATHEMATICS

Competency 0001 Understand number relationships and computational procedures and algorithms.

Fractions, decimals, and percents can be used interchangeably within problems.

→ To change a percent into a decimal, move the decimal point two places to the left and drop off the percent sign.

→ To change a decimal into a percent, move the decimal two places to the right and add on a percent sign.

→ To change a fraction into a decimal, divide the numerator by the denominator.

→ To change a decimal number into an equivalent fraction, write the decimal part of the number as the fraction's numerator. As the fraction's denominator use the place value of the last column of the decimal. Reduce the resulting fraction as far as possible.

Example: J.C. Nickels has Hunch jeans $1/4$ off the usual price of $36.00. Shears and Roadkill have the same jeans 30% off their regular price of $40. Find the cheaper price.

$1/4$ = .25 so .25(36) = $9.00 off $36 - 9 = $27 sale price

30% = .30 so .30(40) = $12 off $40 - 12 = $28 sale price

The price at J.C Nickels is actually lower.

To **convert a fraction to a decimal**, simply divide the numerator (top) by the denominator (bottom). Use long division if necessary.

If a decimal has a fixed number of digits, the decimal is said to be terminating. To write such a decimal as a fraction, first determine what place value the farthest right digit is in, for example: tenths, hundredths, thousandths, ten thousandths, hundred thousands, etc. Then drop the decimal and place the string of digits over the number given by the place value.

If a decimal continues forever by repeating a string of digits, the decimal is said to be repeating. To write a repeating decimal as a fraction, follow these steps.

a. Let $x =$ the repeating decimal (ex. $x = .716716716...$)
b. Multiply x by the multiple of ten that will move the decimal just to the right of the repeating block of digits. (ex. $1000x = 716.716716...$)
c. Subtract the first equation from the second. (ex. $1000x - x = 716.716.716... - .716716...$)
d. Simplify and solve this equation. The repeating block of digits will subtract out. (ex. $999x = 716$ so $x = 716/999$)
e. The solution will be the fraction for the repeating decimal.

To change a number into **scientific notation**, move the decimal point so that only a single digit is to the left of the decimal point. Drop off any trailing zeros. Multiply this number times 10 to a power. The power is the number of positions that the decimal point is moved. The power is negative if the original number is a decimal number between 1 and -1. Otherwise the power is positive.

Example: Change into scientific notation:

$4,380,000,000$	Move decimal behind the 4
4.38	Drop trailing zeros.
$4.38 \times 10^?$	Count positions that the decimal point has moved.
4.38×10^9	This is the answer.
$-.0000407$	Move decimal behind the 4
-4.07	Count positions that the decimal point has moved.
-4.07×10^{-5}	Note negative exponent.

If a number is already in scientific notation, it can be changed back into regular decimal form. If the exponent on the number 10 is negative, move the decimal point to the left that number of places.. If the exponent on the number 10 is positive, move the decimal point to the right.

Example: Change back into decimal form:

3.448×10^{-2} Move decimal point 2 places left, since exponent is negative.

$.03448$ This is the answer.

6×10^4 Move decimal point 4 places right, since exponent is positive.

$60,000$ This is the answer.

To add or subtract in scientific notation, the exponents must be the same. Then add the decimal portions, keeping the power of 10 the same. Then move the decimal point and adjust the exponent to keep the number to the left of the decimal point to a single digit.

Example:

$$\begin{array}{r} 6.22 \times 10^3 \\ + 7.48 \times 10^3 \\ \hline 13.70 \times 10^3 \\ 1.37 \times 10^4 \end{array}$$

Add these as is.

Now move decimal 1 more place to the left and add 1 more exponent.

To multiply or divide in scientific notation, multiply or divide the decimal part of the numbers. In multiplication, add the exponents of 10. In division, subtract the exponents of 10. Then move the decimal point and adjust the exponent to keep the number to the left of the decimal point to a single digit.

Example:

$(5.2 \times 10^5)(3.5 \times 10^2)$ Multiply $5.2 \cdot 3.5$

18.2×10^7 Add exponent

1.82×10^8 Move decimal point and increase the exponent by 1.

Example:

$\dfrac{(4.1076 \times 10^3)}{2.8 \times 10^{-4}}$ Divide 4.1076 by 2.8

Subtract $3 - (^{-}4)$

1.467×10^7

A **ratio** is a comparison of 2 numbers. If a class had 11 boys and 14 girls, the ratio of boys to girls could be written one of 3 ways:

$$11{:}14 \quad \text{or} \quad 11 \text{ to } 14 \quad \text{or} \quad \frac{11}{14}$$

The ratio of girls to boys is:

$$14{:}11,\ 14 \text{ to } 11 \text{ or } \frac{14}{11}$$

Ratios can be reduced when possible. A ratio of 12 cats to 18 dogs would reduce to 2:3, 2 to 3 or $2/3$.

Note: Read ratio questions carefully. Given a group of 6 adults and 5 children, the ratio of children to the entire group would be 5:11.

A **proportion** is an equation in which a fraction is set equal to another. To solve the proportion, multiply each numerator times the other fraction's denominator. Set these two products equal to each other and solve the resulting equation. This is called **cross-multiplying** the proportion.

Example: $\frac{4}{15} = \frac{x}{60}$ is a proportion.

To solve this, cross multiply.

$$(4)(60) = (15)(x)$$
$$240 = 15x$$
$$16 = x$$

Example: $\frac{x+3}{3x+4} = \frac{2}{5}$ is a proportion.

To solve, cross multiply.

$$5(x+3) = 2(3x+4)$$
$$5x + 15 = 6x + 8$$
$$7 = x$$

Example: $\frac{x+2}{8} = \frac{2}{x-4}$ is another proportion.

To solve, cross multiply.

$$(x+2)(x-4) = 8(2)$$
$$x^2 - 2x - 8 = 16$$
$$x^2 - 2x - 24 = 0$$
$$(x-6)(x+4) = 0$$
$$x = 6 \text{ or } x = {}^{-}4$$

Both answers work.

The unit rate for purchasing an item is its price divided by the number of pounds/ounces, etc. in the item. The item with the lower unit rate is the lower price.

<u>Example:</u> Find the item with the best unit price:

$1.79 for 10 ounces
$1.89 for 12 ounces
$5.49 for 32 ounces

$\frac{1.79}{10} = 0.179$ per ounce $\frac{1.89}{12} = 0.1575$ per ounce $\frac{5.49}{32} = 0.172$ per ounce

$1.89 for 12 ounces is the best price.

A second way to find the better buy is to make a proportion with the price over the number of ounces, etc. Cross multiply the proportion, writing the products above the numerator that is used. The better price will have the smaller product.

<u>Example:</u> Find the better buy:

$8.19 for 40 pounds or $4.89 for 22 pounds

Find the unit price.

$$\frac{40}{8.19} = \frac{1}{x} \qquad \frac{22}{4.89} = \frac{1}{x}$$
$$40x = 8.19 \qquad 22x = 4.89$$
$$x = 0.20475 \qquad x = 0.222\overline{27}$$

Since 0.20475 < $0.222\overline{27}$, $8.19 is less and is a better buy.

To find the amount of sales tax on an item, change the percent of sales tax into an equivalent decimal number. Then multiply the decimal number times the price of the object to find the sales tax. The total cost of an item will be the price of the item plus the sales tax.

Example: A guitar costs $120 plus 7% sales tax. How much are the sales tax and the total bill?

7% = .07 as a decimal (.07)(120) = $8.40 sales tax
$120 + $8.40 = $128.40 ← total cost

An alternative method to find the total cost is to multiply the price times the factor 1.07 (price + sales tax):

$$\$120 \times 1.07 = \$8.40$$

This gives you the total cost in fewer steps.

Example: A suit costs $450 plus 6½% sales tax. How much are the sales tax and the total bill?

6½% = .065 as a decimal
(.065)(450) = $29.25 sales tax
$450 + $29.25 = $479.25 ← total cost

An alternative method to find the total cost is to multiply the price times the factor 1.065 (price + sales tax):

$$\$450 \times 1.065 = \$479.25$$

This gives you the total cost in fewer steps.

An **algorithm** is a method of calculating; simply put, it can be multiplication, subtraction, or a combination of operations. When working with computers and calculators we employ **algorithmic thinking**, which means performing mathematical tasks by creating a sequential and often repetitive set of steps. A simple example would be to create an algorithm to generate the Fibonacci numbers utilizing the MR and M+ keys found on most calculators. The table below shows Entry made in the calculator, the value *x* seen in the display, and the value M contained in the memory.

Entry	ON/AC	1	M+	+	M+	MR	+	M+	MR	+	…
x	0	1	1	1	1	2	3	3	5	8	…
M	0	0	1	1	2	2	2	5	5	5	…

This eliminates the need to repeatedly enter required numbers.

Estimation and approximation may be used to check the reasonableness of answers.

Example: Estimate the answer.

$$\frac{58 \times 810}{1989}$$

58 becomes 60, 810 becomes 800 and 1989 becomes 2000.

$$\frac{60 \times 800}{2000} = 24$$

Word problems: An estimate may sometimes be all that is needed to solve a problem.

Example: Janet goes into a store to purchase a CD on sale for $13.95. While shopping, she sees two pairs of shoes, prices $19.95 and $14.50. She only has $50. Can she purchase everything?

Solve by rounding:

$19.95→$20.00
$14.50→$15.00
$13.95→$14.00
$49.00 Yes, she can purchase the CD and the shoes.

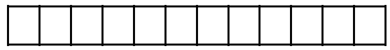
Calculators are an important tool. They should be encouraged in the classroom and at home. They do not replace basic knowledge but they can relieve the tedium of mathematical computations, allowing students to explore more challenging mathematical directions. Students will be able to use calculators more intelligently if they are taught how. Students need to always check their work by estimating. The goal of mathematics is to prepare the child to survive in the real world. Technology is a reality in today's society.

Computers cannot replace teachers. However, they can be used to enhance the curriculum. They may be used cautiously to help students practice basic skills. Many excellent programs exist to encourage higher-order thinking skills, creativity and problem solving. Learning to use technology appropriately is an important preparation for adulthood. Computers can also show the connections between mathematics and the real world.

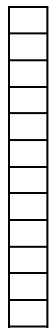
MANIPULATIVES

Example:
Using tiles to demonstrate both geometric ideas and number theory.

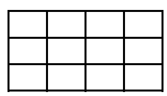
Give each group of students 12 tiles and instruct them to build rectangles. Students draw their rectangles on paper.



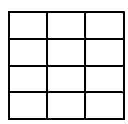
12×1



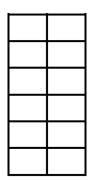
1×12



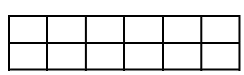
3×4



4×3



6×2



2×6

Encourage students to describe their reactions. Extend to 16 tiles. Ask students to form additional problems.