COMPETENCY 1.0 KNOWLEDGE OF NUMBER SENSE, CONCEPTS, AND OPERATIONS

Skill 1.1 Apply ratio and proportion to solve real-world problems.

A **ratio** is a comparison of 2 numbers. If a class had 11 boys and 14 girls, the ratio of boys to girls could be written one of 3 ways:

11:14 or 11 to 14 or $\frac{11}{14}$

The ratio of girls to boys is:

14:11, 14 to 11 or $\frac{14}{11}$

Ratios can be reduced when possible. A ratio of 12 cats to 18 dogs would reduce to 2:3, 2 to 3 or $2/3$.

Note: Read ratio questions carefully. Given a group of 6 adults and 5 children, the ratio of children to the entire group would be 5:11.

A **proportion** is an equation in which a fraction is set equal to another. To solve the proportion, multiply each numerator times the other fraction's denominator. Set these two products equal to each other and solve the resulting equation. This is called **cross-multiplying** the proportion.

Example: $\frac{4}{15} = \frac{x}{60}$ is a proportion.

To solve this, cross multiply.

$$(4)(60) = (15)(x)$$

$$240 = 15x$$

$$16 = x$$

Example: $\frac{x+3}{3x+4} = \frac{2}{5}$ is a proportion.

To solve, cross multiply.

$5(x+3) = 2(3x+4)$

$5x + 15 = 6x + 8$

$7 = x$

Example: $\frac{x+2}{8} = \frac{2}{x-4}$ is another proportion.

To solve, cross multiply.

$(x+2)(x-4) = 8(2)$

$x^2 - 2x - 8 = 16$

$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$x = 6 \text{ or } x = {}^{-}4$

Proportions can be used to solve word problems whenever relationships are compared. Some situations include scale drawings and maps, similar polygons, speed, time and distance, cost, and comparison shopping.

Example: Which is the better buy, 6 items for \$1.29 or 8 items for \$1.69?

Find the unit price.

$\frac{6}{1.29} = \frac{1}{x}$	$\frac{8}{1.69} = \frac{1}{x}$
6x = 1.29	8x = 1.69
x = 0.215	x = 0.21125

Thus, 6 items for \$1.29 is the better buy.

Example: A car travels 125 miles in 2.5 hours.. How far will it go in 6 hours?

Write a proportion comparing the distance and time.

$$\frac{miles}{hours} \qquad \frac{125}{2.5} = \frac{x}{6}$$

$2.5x = 750$

$x = 300$

Thus, the car can travel 300 miles in 6 hours.

Example: The scale on a map is $\frac{3}{4}$ inch = 6 miles. What is the actual distance between two cities if they are $1\frac{1}{2}$ inches apart on the map?

Write a proportion comparing the scale to the actual distance.

scale actual

$$\frac{\frac{3}{4}}{1\frac{1}{2}} = \frac{6}{x}$$

$\frac{3}{4}x = 1\frac{1}{2} \times 6$

$\frac{3}{4}x = 9$

$x = 12$

Thus, the actual distance between the cities is 12 miles.

Skill 1.2 Solve real-world problems that involve percents, decimals, fractions, or numbers expressed in scientific and exponential notation.

Percent = per 100 (written with the symbol %). Thus 10% $= \frac{10}{100} = \frac{1}{10}$.

Decimals = deci = part of ten. To find the decimal equivalent of a fraction, use the denominator to divide the numerator as shown in the following example.

Example: Find the decimal equivalent of $\frac{7}{10}$.

Since 10 cannot divide into 7 evenly

$$\frac{7}{10} = 0.7$$

The **exponent form** is a shortcut method to write repeated multiplication. Basic form: b^n, where *b* is called the base and *n* is the exponent. *b* and *n* are both real numbers. b^n implies that the base *b* is multiplied by itself *n* times.

Examples: $3^4 = 3 \times 3 \times 3 \times 3 = 81$

$2^3 = 2 \times 2 \times 2 = 8$

$(^-2)^4 = (^-2) \times (^-2) \times (^-2) \times (^-2) = 16$

$^-2^4 =^- (2 \times 2 \times 2 \times 2) =^- 16$

Key exponent rules:

For '*a*' nonzero, and '*m*' and '*n*' real numbers:

1) $a^m \cdot a^n = a^{(m+n)}$ Product rule

2) $\dfrac{a^m}{a^n} = a^{(m-n)}$ Quotient rule

3) $\dfrac{a^{^-m}}{a^{^-n}} = \dfrac{a^n}{a^m}$

When 10 is raised to any power, the exponent tells the numbers of zeroes in the product.

Example: $10^7 = 10{,}000{,}000$

Caution: Unless the negative sign is inside the parentheses and the exponent is outside the parentheses, the sign is not affected by the exponent.

$(^-2)^4$ implies that -2 is multiplied by itself 4 times.

$^-2^4$ implies that 2 is multiplied by itself 4 times, then the answer is negated.

Scientific notation is a more convenient method for writing very large and very small numbers. It employs two factors. The first factor is a number between 1 and 10. The second factor is a power of 10. This notation is a "shorthand" for expressing large numbers (like the weight of 100 elephants) or small numbers (like the weight of an atom in pounds).

Recall that:

$10^n = (10)^n$ Ten multiplied by itself *n* times.

$10^0 = 1$ Any nonzero number raised to power of zero is 1.

$10^1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1000$ (kilo)

$10^{-1} = 1/10$ (deci)

$10^{-2} = 1/100$ (centi)

$10^{-3} = 1/1000$ (milli)

$10^{-6} = 1/1{,}000{,}000$ (micro)

Example: Write 46,368,000 in scientific notation.

1) Introduce a decimal point and decimal places.
46,368,000 = 46,368,000.0000

2) Make a mark between the two digits that give a number between -9.9 and 9.9.
$4 \wedge 6{,}368{,}000$.0000

3) Count the number of digit places between the decimal point and the $\wedge$ mark. This number is the '*n*'-the power of ten.

So, 46,368,000 $= 4.6368 \times 10^7$

Example: Write 0.00397 in scientific notation.

1) Decimal place is already in place.

2) Make a mark between 3 and 9 to get a one number between -9.9 and 9.9.

3) Move decimal place to the mark (3 hops).

$0.003 \wedge 97$

Motion is to the right, so *n* of 10^n is negative.

Therefore, $0.00397 = 3.97 \times 10^{-3}$

Word problems involving percents can be solved by writing the problem as an equation, then solving the equation. Keep in mind that **"of" means "multiplication**" and **"is" means "equals."**

Example: The Ski Club has 85 members. Eighty percent of the members are able to attend the meeting. How many members attend the meeting?

Restate the problem. What is 80% of 85?
Write an equation. $n = 0.8 \times 85$
Solve. $n = 68$

Sixty-eight members attend the meeting.

Example: There are 64 dogs in the kennel. Forty-eight are collies. What percent are collies?

Restate the problem. 48 is what percent of 64?
Write an equation. $48 = n \times 64$
Solve. $\frac{48}{64} = n$
$n = \frac{3}{4} = 75\%$

75% of the dogs are collies.

Example: The auditorium was filled to 90% capacity. There were 558 seats occupied. What is the capacity of the auditorium?

Restate the problem. 90% of what number is 558?

Write an equation. $0.9n = 558$

Solve. $n = \frac{558}{.9}$

$n = 620$

The capacity of the auditorium is 620 people.

Example: Shoes cost $42.00. Sales tax is 6%. What is the total cost of the shoes?

Restate the problem. What is 6% of 42?

Write an equation. $n = 0.06 \times 42$

Solve. $n = 2.52$

Add the sales tax to the cost. $42.00 + $2.52 = $44.52

The total cost of the shoes, including sales tax, is $44.52.

An alternative method would be to multiply $42.00 by 1.06.

$42.00 x 1.06 = $44.52 (cost including sales tax)

COMMON EQUIVALENTS

$\frac{1}{2} = 0.5 = 50\%$

$\frac{1}{3} = 0.333 = 33\frac{1}{3}\%$

$\frac{1}{4} = 0.25 = 25\%$

$\frac{1}{5} = 0.2 = 20\%$

$\frac{1}{6} = 0.1667 = 16\frac{2}{3}\%$

$\frac{1}{8} = 0.125 = 12\frac{1}{2}\%$

$\frac{1}{10} = 0.1 = 10\%$

$\frac{2}{3} = 0.6667 = 66\frac{2}{3}\%$

$\frac{5}{6} = 0.833 = 83\frac{1}{3}\%$

$\frac{3}{8} = 0.375 = 37\frac{1}{2}\%$

$\frac{5}{8} = 0.625 = 62\frac{1}{2}\%$

$\frac{7}{8} = 0.875 = 87\frac{1}{2}\%$

$1 = 1.0 = 100\%$

Skill 1.3 Apply number concepts including primes, factors, and multiples to build number sequences.

A numeration system is a set of numbers represented a by a set of symbols (numbers, letters, or pictographs). Sets can have different bases of numerals within the set. Instead of our base 10, a system may use any base set from 2 on up. The position of the number in that representation defines its exact value. Thus, the numeral 1 has a value of ten when represented as "10". Early systems, such as the Babylonian, used position in relation to other numerals or column position for this purpose since they lacked a zero to represent an empty position.

A base of 2 uses only 0 and 1.

Decimal Binary Conversion		
Decimal	Binary	Place Value
1	1	2^0
2	10	2^1
4	100	2^2
8	1000	2^3

Thus, 9 in Base 10 becomes 1001 in Base 2.

9+4 = 13 (Base 10) becomes 1001 + 100 = 1101 (Base 2).
Fractions, ratios and other functions alter in the same way.
Computers use a base of 2 but combine it into 4 units called a byte to function in base 16 (hexadecimal). A base of 8 (octal) was also used by older computers.

Prime numbers are numbers that can only be factored into 1 and the number itself. When factoring into prime factors, all the factors must be numbers that cannot be factored again (without using 1). Initially numbers can be factored into any 2 factors. Check each resulting factor to see if it can be factored again. Continue factoring until all remaining factors are prime. This is the list of prime factors. Regardless of which way the original number was factored, the final list of prime factors will always be the same.

Example: Factor 30 into prime factors.

Divide by 2 as many times as you can, then by 3, then by other successive primes as required.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Factor 30 into any 2 factors.

$5 \cdot 6$ Now factor the 6.
$5 \cdot 2 \cdot 3$ These are all prime factors.

Factor 30 into any 2 factors.

$3 \cdot 10$ Now factor the 10.
$3 \cdot 2 \cdot 5$ These are the same prime factors even though the original factors were different.

Example: Factor 240 into prime factors.

Factor 240 into any 2 factors.

$24 \cdot 10$ Now factor both 24 and 10.
$4 \cdot 6 \cdot 2 \cdot 5$ Now factor both 4 and 6.
$2 \cdot 2 \cdot 2 \cdot 3 \cdot 2 \cdot 5$ These are prime factors.

This can also be written as $2^4 \cdot 3 \cdot 5$