

**COMPETENCY 1 KNOWLEDGE OF MATHEMATICS THROUGH PROBLEM SOLVING**

**Skill 1.1 Identify appropriate mathematical problems from real-world situations.**

The unit rate for purchasing an item is its price divided by the number of ounces, pounds, and gallons etc. of the item. The item with the lower unit rate has the lower price.

Example: Find the item with the lowest unit price:

\$1.79 for 10 ounces  
\$1.89 for 12 ounces  
\$5.49 for 32 ounces

$$\frac{\$1.79}{10\text{oz.}} = \$0.179/\text{oz.} \quad \frac{\$1.89}{12\text{oz.}} = \$0.1575/\text{oz.} \quad \frac{\$5.49}{32\text{oz.}} = \$0.172/\text{oz.}$$

\$1.89 for 12 ounces is the best price.

$$\frac{40}{8.19} = \frac{1}{x}$$

$$40x = 8.19$$

$$x = 0.20475$$

$$\frac{22}{4.89} = \frac{1}{x}$$

$$22x = 4.89 < 0.2222\overline{7} \text{ Note: Unit}$$

$$x = 0.2222\overline{7}$$

prices can also be calculated by setting up proportions (see Skill 5.10).

To find the amount of sales tax on an item, change the percent of sales tax into an equivalent decimal number by moving the decimal point two places to the left. Then multiply the decimal number by the price of the object to find the sales tax. The total cost of an item will be the price of the item plus the sales tax.

Example: A guitar costs \$120 plus 7% sales tax. How much are the sales tax and the total bill?

$$\begin{aligned}7\% &= 0.07 \text{ in decimal form} \\(0.07)(\$120) &= \$8.40 \text{ sales tax} \\\$120 + \$8.40 &= \$128.40 \leftarrow \text{total cost}\end{aligned}$$

An alternative method to find the total cost is to multiply the price times the factor 1.07 (price + sales tax):

$$\$120 \times 1.07 = \$128.40$$

This gives you the total cost in fewer steps.

Example: A suit costs \$450 plus 6½% sales tax. How much are the sales tax and the total bill?

$$\begin{aligned}6\frac{1}{2}\% &= 0.065 \text{ in decimal form} \\(0.065)(450) &= \$29.25 \text{ sales tax} \\\$450 + \$29.25 &= \$479.25 \leftarrow \text{total cost}\end{aligned}$$

Using the alternative method to find total cost, multiply the price times the factor 1.065 (price + sales tax):

$$\$450 \times 1.065 = \$479.25$$

This gives you the total cost in fewer steps.

**Skill 1.2 Apply problem-solving strategies to solve non-routine problems with multiple steps.**

Teachers can promote problem solving by allowing multiple attempts at problems. There are several specific problem solving skills with which teachers should be familiar.

The **guess-and-check** strategy calls for students to make an initial guess at the solution, check the answer, and use the outcome of to guide the next guess. With each successive guess, the student should get closer to the correct answer. Constructing a table from the guesses can help organize the data.

Example: There are 100 coins in a jar. Ten are dimes. The rest are pennies and nickels. There are twice as many pennies as nickels. How many pennies and nickels are in the jar?

There are 90 total nickels and pennies in the jar (100 coins – 10 dimes).

There are twice as many pennies as nickels. Make guesses that fulfill the criteria and adjust based on the answer found. Continue until we find the correct answer, 60 pennies and 30 nickels.

Number of Pennies	Number of Nickels	Total Number of Pennies and Nickels
40	20	60
80	40	120
70	35	105
60	30	90

When solving a problem where the final result and the steps to reach the result are given, students must **work backwards** to determine what the starting point must have been.

Example: John subtracted 7 from his age and divided the difference by 3. The final result was 4. What is John's age?

Work backward by reversing the operations.

$$4 \times 3 = 12$$

$$12 + 7 = 19$$

John is 19 years old.

**Skill 1.3 Evaluate the reasonableness of results with respect to the original problem.**

Estimation and approximation may be used to check the reasonableness of answers. (See Skill 5.2.)

**Skill 1.4 Use mathematics to solve problems in other contexts.**

Elapsed time problems are usually one of two types. One type of problem is the elapsed time between two times given in hours, minutes and seconds. The other common type of problem is between two times given in months and years.

For any time of day past noon, change to military time by adding 12 hours. For instance, 1:15 p.m. would be 13:15. Remember when you borrow a minute or an hour in a subtraction problem that you have borrowed 60 more seconds or minutes.

Example: Find the time from 11:34:22 a.m. until 3:28:40 p.m.

Convert 3:28:40 p.m. to military time

3:28:40

+12:00:00

15:28:40

Now subtract

15:28:40

-11:34:22

:18

Borrow an hour and add 60 more minutes. Subtract.

14:88:40

- 11:34:22

3:54:18 ↔ 3 hours, 54 minutes, 18 seconds

Example: John lived in Arizona from September 1991 until March 1995. How long is that?

	year	month
March 1995	= 95	03
September 1991	= <u>-91</u>	<u>09</u>

Borrow a year and convert it into 12 more months, subtract.

	year	month
March 1995	= 94	15
September 1991	= <u>-91</u>	<u>09</u>
	3 yrs 6 months	

Example: A race took the winner 1 hr. 58 min. 12 sec. on the first half of the race and 2 hr. 9 min. 57 sec. on the second half of the race. How much time did the entire race take?

$$\begin{array}{r} 1 \text{ hr. } 58 \text{ min. } 12 \text{ sec.} \\ + 2 \text{ hr. } 9 \text{ min. } 57 \text{ sec.} \quad \text{Add} \\ \hline 3 \text{ hr. } 67 \text{ min. } 69 \text{ sec.} \\ + 1 \text{ min. } -60 \text{ sec.} \quad \text{Convert 60 sec. to 1 min.} \\ \hline 3 \text{ hr. } 68 \text{ min. } 9 \text{ sec.} \\ + 1 \text{ hr. } -60 \text{ min.} \quad \text{Convert 60 min. to 1 hr.} \\ \hline 4 \text{ hr. } 8 \text{ min. } 9 \text{ sec.} \quad \leftarrow \text{Final answer} \end{array}$$

## COMPETENCY 2 KNOWLEDGE OF MATHEMATICAL REPRESENTATIONS

### Skill 2.1 Identify appropriate representations or models for mathematics operations or situations using written, concrete, pictorial, graphical, or algebraic methods.

According to Piaget, there are four primary cognitive structures or development stages: sensorimotor, preoperations, concrete operations, and formal operations. In the sensorimotor stage (0-2 years), intelligence takes the form of motor actions. In the preoperation stage (3-7 years), intelligence is intuitive in nature. Intelligence in the concrete operational stage (8-11 years) is logical but depends upon concrete referents. In the final stage of formal operations (12-15 years), thinking involves abstractions.

Mathematical operations include addition, subtraction, multiplication and division.

Addition can be indicated by the expressions: sum, greater than, and, more than, increased by, added to.

Subtraction can be expressed by: difference, fewer than, minus, less than, decreased by, subtracted from.

Multiplication is shown by: product, times, multiplied by, twice.

Division is used for: quotient, divided by, ratio.

<u>Examples:</u>	7 added to a number	$n + 7$
	a number decreased by 8	$n - 8$
	12 times a number divided by 7	$12n \div 7$
	28 less than a number	$n - 28$
	the ratio of a number to 55	$\frac{n}{55}$
	4 times the sum of a number and 21	$4(n + 21)$

Mathematical operations can be shown using manipulatives or drawings.

Multiplication can be shown using arrays.

$3 \times 4$  may be expressed as 3 rows of 4 each

□ □ □ □  
□ □ □ □  
□ □ □ □

Addition and subtractions can be demonstrated with symbols.

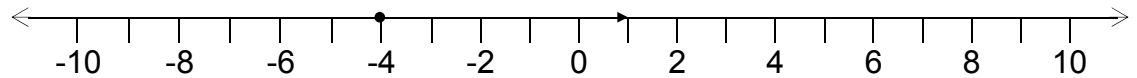
$$\begin{array}{c} \psi \ \psi \ \psi \ \xi \ \xi \ \xi \ \xi \\ 3 + 4 = 7 \\ 7 - 3 = 4 \end{array}$$

Fractions can be clarified using pattern blocks, fraction bars, or paper folding.

**Skill 2.2 Interpret results as illustrated by the use of mathematical representations.**

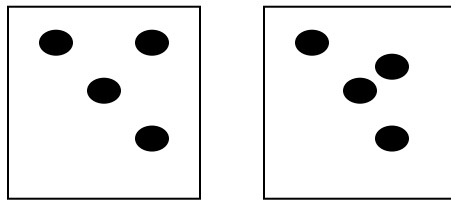
Students of mathematics must be able to recognize and interpret the different representations of arithmetic operations. Diagrams of arithmetic operations can present mathematical data in visual form.

For example, a number line can be used to add and subtract.

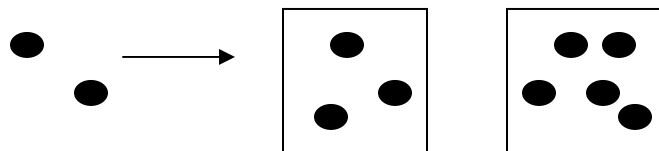


Five added to negative four on the number line or  $-4 + 5 = 1$ .

Pictorial representations can also be used to explain the arithmetic processes.



Two groups of four equal eight or  $2 \times 4 = 8$  shown in picture form.



Adding two objects to three or  $2 + 3 = 5$  shown in picture form.

**Skill 2.3**     **Select appropriate manipulatives, mathematical models, or technology for teaching particular mathematics concepts (e.g., tiles for teaching area, graphing calculators for teaching algebra).**

The use of supplementary materials in the classroom can greatly enhance the learning experience by stimulating student interest and satisfying different learning styles. Manipulatives, models, and technology are examples of tools available to teachers.

**Manipulatives** are materials that students can physically handle and move. Manipulatives allow students to understand mathematic concepts by allowing them to see concrete examples of abstract processes. Manipulatives are attractive to students because they appeal to the students' visual and tactile senses. Available for all levels of math, manipulatives are useful tools for reinforcing operations and concepts. They are not, however, a substitute for the development of sound computational skills.

**Models** are another means of representing mathematical concepts by relating the concepts to real-world situations. Teachers must choose wisely when devising and selecting models because, to be effective, models must be applied properly. For example, a building with floors above and below ground is a good model for introducing the concept of negative numbers. It would be difficult, however, to use the building model in teaching subtraction of negative numbers.

Finally, there are many forms of **technology** available to math teachers. For example, students can test their understanding of math concepts by working on skill specific computer programs and websites. Graphing calculators can help students visualize the graphs of functions. Teachers can also enhance their lectures and classroom presentations by creating multimedia presentations.

Technology is increasingly important in schools today. Knowing what each item is, how to use each item and the importance of each item beyond the walls of the school is extremely important for teachers and students. We will cover some of the most important technology tools below:

- **Computers:** Personal computing, word processing, research, and multi-media devices that utilize hard drive or "memory" in order to save, transform, and process information. It is easy to look past the importance of the computer, itself, in the wake of internet technology. However, many children from low socioeconomic backgrounds throughout the country lack access to computers and may not have basic computing skills. Indeed, many teachers must be taught to utilize some of the more advanced features that could help them with their daily work. In general, the computer is viewed as a tool for storage and efficient processing.
- **Internet:** Platform for the sharing and organization of information throughout the world. It began with simple phone connections between dozens of computers in the 70s and 80s. In the 90s, it became a standard feature on virtually every computer. Through websites, the internet allows people to view, post, and find anything that is available for public viewing. This is helpful when it comes to classroom research, however, there is plenty of material that is inappropriate for children and irrelevant to school. Care should be given to filtering out websites that are inappropriate for students.
- **Video Projection:** Video in the classroom is a common tool to enhance the learning of students. Indeed, video has become a "text" in itself, much like literature. While the internet has been a good source for information, video continues to be a great source of refined, carefully structured information for teachers and students. In addition, it is a great way for students to express themselves, as they develop and present multi-media presentations, for example.
- **CDs or DVDs:** Data discs that are inserted into computers. Usually, CDs and DVDs are discs contain computer programs. It used to be that they contained information, such as encyclopedic information. Now, that material is typically stored on a computer hard drives or servers.

While computers cannot replace teachers, they can be used to enhance the curriculum. Computers may be used to help students practice basic skills. Many excellent programs exist to encourage higher-order thinking skills, creativity and problem solving. Learning to use technology appropriately is an important preparation for adulthood. Computers can also show the connections between mathematics and the real world.

Calculators: Handheld mathematical computation devices. Usually, these help students and teachers perform basic mathematical functions. More advanced functions can be performed by various computer programs.

### Manipulatives

Using tiles to demonstrate both geometric ideas and number theory.

Example: Using 12 tiles, build rectangles of the same area but different perimeters.

