SUBAREA I. MATHEMATICAL PROCESSES, METHODS, NUMBER CONCEPTS, AND THEIR HISTORICAL DEVELOPMENT

COMPETENCY 0001 UNDERSTANDS PROBLEM-SOLVING STRATEGIES, CONNECTIONS AMONG DIFFERENT MATHEMATICAL IDEAS, AND THE USE OF MATHEMATICAL MODELING TO SOLVE REAL-WORLD PROBLEMS.

Estimation and testing for **reasonableness** are related skills students should employ prior to and after solving a problem. These skills are particularly important when students use calculators to find answers.

Example:

Find the sum of 4387 + 7226 + 5893.

4300 + 7200 + 5800 = 17300	Estimation.
4387 + 7226 + 5893 = 17506	Actual sum.

By comparing the estimate to the actual sum, students can determine that their answer is reasonable.

Problem solving strategies are simply plans of attack. Student often panic when confronted with word problems. If they have a "list" of ideas, ways to attempt a solution, they will be able to approach the problems more calmly and confidently.

Successful math teachers introduce their students to multiple problem solving strategies and create a classroom environment where free thought and experimentation are encouraged. Teachers can promote problem solving by allowing multiple attempts at problems, giving credit for reworking test or homework problems, and encouraging the sharing of ideas through class discussion. There are several specific problem solving skills with which teachers should be familiar.

The **guess-and-check** strategy calls for students to make an initial guess at the solution, check the answer, and use the outcome as a guide for the next guess. With each successive guess, the student should get closer to the correct answer. Constructing a table from the guesses can help organize the data.

Example:

There are 100 coins in a jar. 10 are dimes. The rest are pennies and nickels. There are twice as many pennies as nickels. How many pennies and nickels are in the jar?

There are 90 total nickels and pennies in the jar (100 coins – 10 dimes).

There are twice as many pennies as nickels. Make guesses that fulfill the criteria and adjust based on the answer found. Continue until we find the correct answer, 60 pennies and 30 nickels.

Number of Pennies	Number of Nickels	Total Number of Pennies and Nickels
40	20	60
80	40	120
70	35	105
60	30	90

When solving a problem where the final result and the steps to reach the result are given, students must **work backwards** to determine what the starting point must have been.

Example:

John subtracted seven from his age, and divided the result by 3. The final result was 4. What is John's age?

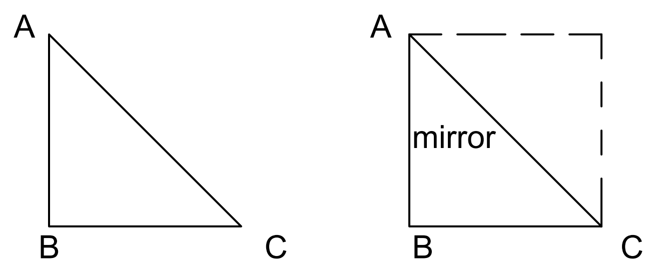
> Work backward by reversing the operations.
> 4 x 3 = 12;
> 12 + 7 = 19
> John is 19 years old.

Some methods include, but are not limited to, draw a diagram, work backwards, guess and check, and solve a simpler problem.

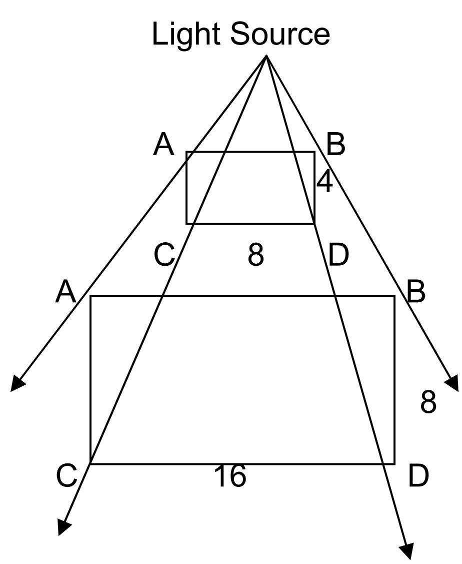
It is helpful to have students work in groups. Mathematics does not have to be a solitary activity. Cooperative learning fosters enthusiasm. Creating their own problems is another useful tool. Also, encourage students to find more than one way to solve a problem. Thinking about problem solving after the solution has been discovered encourages understanding and creativity. The more they practice problems, the more comfortable and positive students will feel.

Reflection devices and other technologies, like overhead projectors, transform geometric constructions in predictable ways. Students should have the ability to recognize the patterns and properties of geometric constructions made with these technologies.

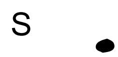
The most common reflection device is a mirror. Mirrors reflect geometric constructions across a given axis. For example, if we place a mirror on the side AC of the triangle (below) the composite image created by the original figure and the reflection is a square.



Projection devices, like overhead projectors, often expand geometric figures. These expansions are proportional, meaning the ratio of the measures of the figure remains the same. Consider the following projection of the rectangle ABCD.

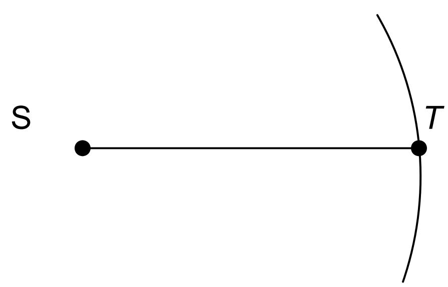


A geometric construction is a drawing made using only a compass and straightedge. A construction consists of only segments, arcs, and points. The easiest construction to make is to duplicate a given line segment. Given segment *AB*, construct a segment equal in length to segment AB by following these steps.



1. Place a point anywhere in the plane to Anchor the duplicate segment. Call this point S.

2. Open the compass to match the length of segment *AB*. Keeping the compass rigid, swing an arc from S.

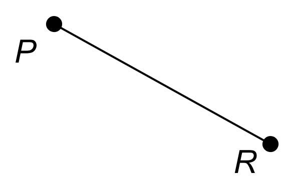


3. Draw a segment from S to any point on the arc. This segment will be the same length as *AB.*

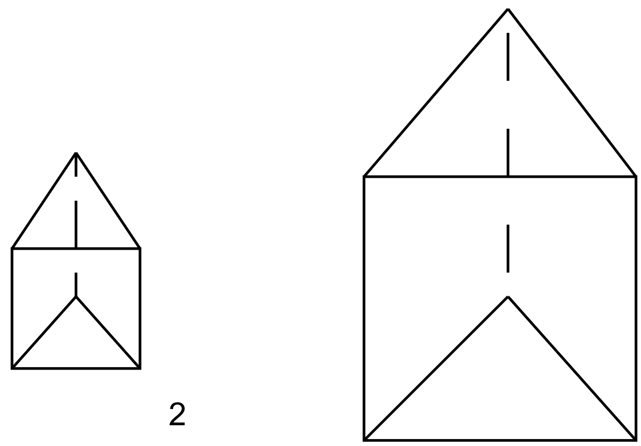
Samples:

Construct segments congruent to the given segments.

1. 

2. 

Similar solids share the same shape but are not necessarily the same size. The ratio of any two corresponding measurements of similar solids is the scale factor. For example, the scale factor for two square pyramids, one with a side measuring 2 inches and the other with a side measuring 4 inches, is 2:4.



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The base perimeter, the surface area, and the volume of similar solids are directly related to the scale factor. If the scale factor of two similar solids is a:b, then the…

ratio of base perimeters = a:b
ratio of areas = $a^2:b^2$
ratio of volumes = $a^3:b^3$

Thus, for the above example the…

ratio of base perimeters = 2:4
ratio of areas = $2^2:4^2 = 4:16$
ratio of volumes = $2^3:4^3 = 8:64$

Sample Problem:

1. What happens to the volume of a square pyramid when the length of the sides of the base are doubled?

scale factor = a:b = 1:2

ratio of volume = $1^3:2^3$ = 1:8 (The volume is increased 8 times.)

Artists, musicians, scientists, social scientists, and business people use mathematical modeling to solve problems in their disciplines. These disciplines rely on the tools and symbology of mathematics to model natural events and manipulate data.

Mathematics is a key aspect of visual art. Artists use the geometric properties of shapes, ratios, and proportions in creating paintings and sculptures. For example, mathematics is essential to the concept of perspective. Artists must determine the appropriate lengths and heights of objects to portray three-dimensional distance in two dimensions.

Mathematics is also an important part of music. Many musical terms have mathematical connections. For example, the musical octave contains twelve notes and spans a factor of two in frequency. In other words, the frequency, the speed of vibration that determines tone and sound quality, doubles from the first note in an octave to the last. Thus, starting from any note we can determine the frequency of any other note with the following formula.

$$\text{Freq} = \text{note} \times 2^{N/12}$$

Where N is the number of notes from the starting point and note is the frequency of the starting note. Mathematical understanding of frequency plays an important role in the tuning of musical instruments.

In addition to the visual and auditory arts, mathematics is an integral part of most scientific disciplines. The uses of mathematics in science are almost endless. The following are but a few examples of how scientists use mathematics. Physical scientists use vectors, functions, derivatives, and integrals to describe and model the movement of objects. Biologists and ecologists use mathematics to model ecosystems and study DNA. Finally, chemists use mathematics to study the interaction of molecules and to determine proper amounts and proportions of reactants.

Many social science disciplines use mathematics to model and solve problems. Economists, for example, use functions, graphs, and matrices to model the activities of producers, consumers, and firms. Political scientists use mathematics to model the behavior and opinions of the electorate. Finally, sociologists use mathematical functions to model the behavior of humans and human populations.

Finally, mathematical problem solving and modeling is essential to business planning and execution. For example, businesses rely on mathematical projections to plan business strategy. Additionally, stock market analysis and accounting rely on mathematical concepts.