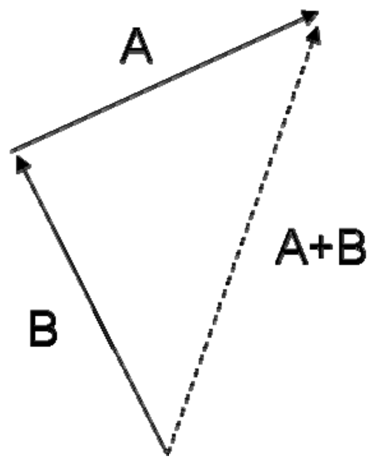
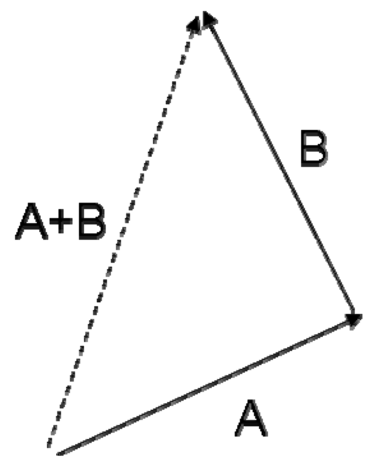
DOMAIN I. MECHANICS

Skill 1 Vectors (properties; addition and subtraction)

Vector space is a collection of objects that have **magnitude** and **direction**. They may have mathematical operations, such as addition, subtraction, and scaling, applied to them. Vectors are usually displayed in boldface or with an arrow above the letter. They are usually shown in graphs or other diagrams as arrows. The length of the arrow represents the magnitude of the vector while the direction in which the arrow points shows the vector direction.

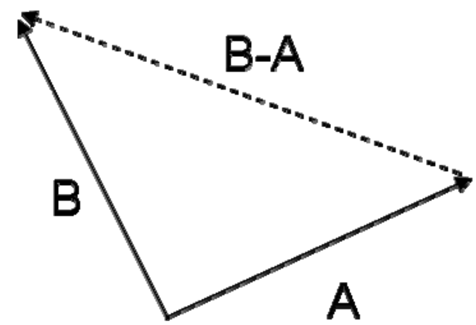
To **add two vectors** graphically, the base of the second vector is drawn from the point of the first vector as shown below with vectors **A** and **B**. The sum of the vectors is drawn as a dashed line, from the base of the first vector to the tip of the second. As illustrated, the order in which the vectors are connected is not significant as the endpoint is the same graphically whether **A** connects to **B** or **B** connects to **A**. This principle is sometimes called the parallelogram rule.





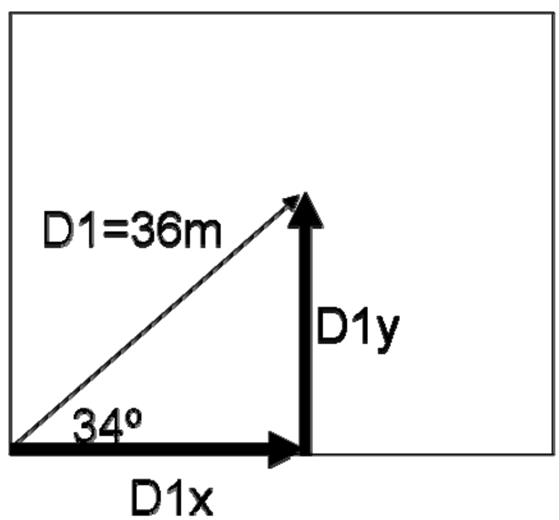
If more than two vectors are to be combined, additional vectors are simply drawn in accordingly with the sum vector connecting the base of the first to the tip of the final vector.

Subtraction of two vectors can be geometrically defined as follows. To subtract **A** from **B**, place the ends of **A** and **B** at the same point and then draw an arrow from the tip of **A** to the tip of **B**. That arrow represents the vector **B-A**, as illustrated below:



To add two vectors without drawing them, the vectors must be broken down into their orthogonal components using sine, cosine, and tangent functions. Add both x components to get the total x component of the sum vector, then add both y components to get the y component of the sum vector. Use the Pythagorean Theorem and the three trigonometric functions to the get the size and direction of the final vector.

<u>Example</u>: Here is a diagram showing the x and y-components of a vector D1:



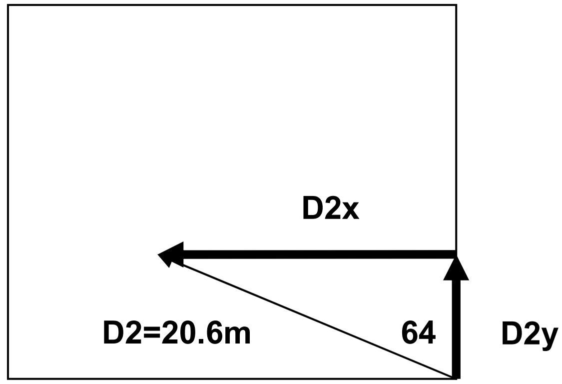
Notice that the x-component D1x is adjacent to the angle of 34 degrees.

Thus D1x=36m (cos34) =29.8m

The y-component is opposite to the angle of 34 degrees.

Thus D1y =36m (sin34) = 20.1m

A second vector D2 is broken up into its components in the diagram below using the same techniques. We find that D2y=9.0m and D2x=-18.5m.



Next we add the x components and the y components to get

DTotal x =11.3 m *and* DTotal y =29.1 m

Now we have to use the Pythagorean theorem to get the total magnitude of the final vector. And the arctangent function to find the direction. As shown in the diagram below.

DTotal=31.2m

tan θ= DTotal y / DTotal x = 29.1m / 11.3 =2.6 θ=69 degrees

Skill 2 Vector multiplication (dot and cross product)

The dot product is also known as the scalar product. This is because the dot product of two vectors is not a vector, but a scalar (i.e., a real number without an associated direction). The definition of the dot product of the two vectors **a** and **b** is:

$$a \bullet b = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n$$

The following is an example calculation of the dot product of two vectors:

[1 3 -5] · [4 -2 -2] = (1)(4) + (3)(-2) + (-5)(-2) = 8

Note that the product is a simple scalar quantity, not a vector. The dot product is commutative and distributive.

Unlike the dot product, the cross product does return another vector. The vector returned by the cross product is orthogonal to the two original vectors. The cross product is defined as:

$$\mathbf{a} \times \mathbf{b} = n\,|\mathbf{a}|\,|\mathbf{b}|\sin\theta$$

where n is a unit vector perpendicular to both **a** and **b** and θ is the angle between **a** and **b**. In practice, the cross product can be calculated as explained below:

Given the orthogonal unit vectors **i**, ,**j**, and **k**, the vector **a** and **b** can be expressed:

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$
$$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$$

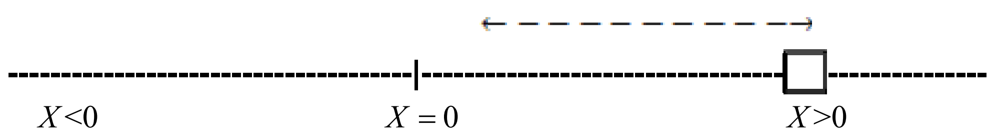
Then we can calculate that

$$\mathbf{a} \times \mathbf{b} = \mathbf{i}(a_2b_3)+\mathbf{j}(a_3b_1)+\mathbf{k}(a_1b_2)-\mathbf{i}(a_3b_2)-\mathbf{j}(a_1b_3)-\mathbf{k}(a_2b_1)$$

The cross product is anticommutative (that is, **a** x **b**= - **b** x **a**) and distributive over addition.

Skill 3 Motion along a straight line (displacement, velocity, acceleration)

Kinematics is the part of mechanics that seeks to understand the motion of objects, particularly the relationship between position, velocity, acceleration and time.



The above figure represents an object and its displacement along one linear dimension.

First we will define the relevant terms:

1. Position or Distance is usually represented by the variable *x*. It is measured relative to some fixed point or datum called the origin in linear units, meters, for example.

***2.* Displacement** is defined as the change in position or distance which an object has moved and is represented by the variables D, d or Δx. Displacement is a vector with a magnitude and a direction.

3. **Velocity** is a vector quantity usually denoted with a V or v and defined as the rate of change of position. Typically units are distance/time, m/s for example. Since velocity is a vector, if an object changes the direction in which it is moving it changes its velocity even if the speed (the scalar quantity that is the magnitude of the velocity vector) remains unchanged.

i) Average velocity: $\underset{v}{\rightarrow} \equiv \frac{\Delta d}{\Delta t} = d_1 - d_0 / {}_{t_1 - t_0}$

The ratio $\Delta d / \Delta t$ is called the average velocity. Average here denotes that this quantity is defined over a period Δt.

ii) Instantaneous velocity is the velocity of an object at a particular moment in time. Conceptually, this can be imagined as the extreme case when Δt is infinitely small.

5. Acceleration represented by *a* is defined as the rate of change of velocity and the units are m/s^2. Both an average and an instantaneous acceleration can be defined similarly to velocity.

From these definitions we develop the kinematic equations. In the following, subscript i denotes initial and subscript f denotes final values for a time period. Acceleration is assumed to be constant with time.

$$v_f = v_i + at \qquad (1)$$

$$d = v_i t + \frac{1}{2} a t^2 \qquad (2)$$

$$v_f^{\,2} = v_i^{\,2} + 2ad \qquad (3)$$

$$d = \left(\frac{v_i + v_f}{2} \right) t \qquad (4)$$

Example:
Leaving a traffic light a man accelerates at 10 m/s^2. a) How fast is he going when he has gone 100 m? b) How fast is he going in 4 seconds? C) How far does he travel in 20 seconds.

Solution**:**
a) Use equation 3. He starts from a stop so v_i=0 and v_f^2=2 x 10m/s^2 x 100m=2000 m^2/s^2 and v_f=45 m/s.
b) Use equation 1. Initial velocity is again zero so v_f=10m/s^2 x 4s=40 m/s.
c) Use equation 2. Since initial velocity is again zero, d=1/2 x 10 m/s^2 x $(20s)^2$=2000 m

Skill 4 Motion in two dimensions (projectile motion, uniform circular motion)

In the previous section, we discussed the relationships between distance, velocity, acceleration and time and the four simple equations that relate these quantities when acceleration is constant (e.g. in cases such as gravity). In two dimensions the same relationships apply, but each dimension must be treated separately.

The most common example of an object moving in two dimensions is a projectile. A projectile is an object upon which the only force acting is gravity. Some examples:
i) An object dropped from rest.
ii) An object thrown vertically upwards at an angle
iii) A canon ball.

Once a projectile has been put in motion (say, by a canon or hand) the only force acting it is gravity, which near the surface of the earth implies it experiences $a=g=9.8m/s^2$.

This is most easily considered with an example such as the case of a bullet shot horizontally from a standard height at the same moment that a bullet is dropped from exactly the same height. Which will hit the ground first? If we assume wind resistance is negligible, then the acceleration due to gravity is our only acceleration on either bullet and we must conclude that they will hit the ground at the same time. The horizontal motion of the bullet is not affected by the downward acceleration.

Example:
I shoot a projectile at 1000 m/s from a perfectly horizontal barrel exactly 1 m above the ground. How far does it travel before hitting the ground?

Solution:
First figure out how long it takes to hit the ground by analyzing the motion in the vertical direction. In the vertical direction, the initial velocity is zero so we can rearrange kinematic equation 2 from the previous section to give:

$t=\sqrt{\frac{2d}{a}}$. Since our displacement is 1 m and $a=g=9.8m/s^2$, t=0.45 s.

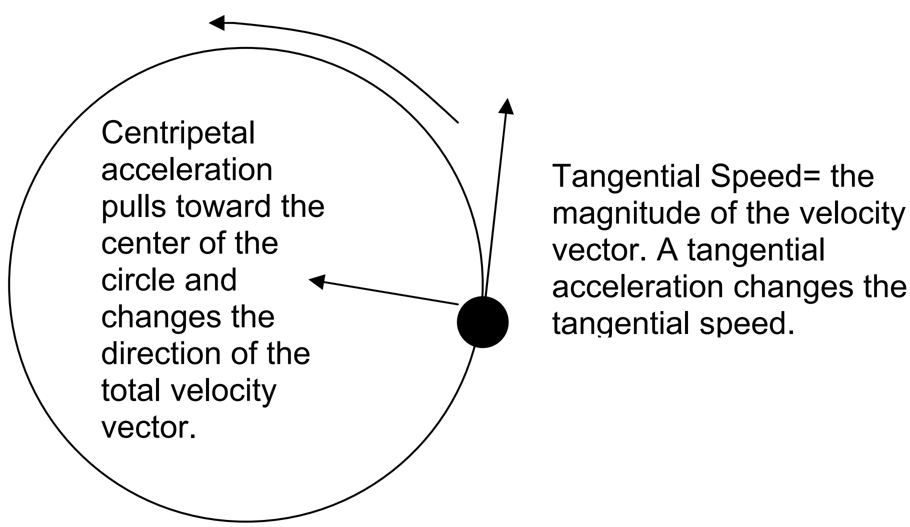
Now use the time to hitting the ground from the previous calculation to calculate how far it will travel horizontally. Here the velocity is 1000m/s and there is no acceleration. So we simple multiply velocity with time to get the distance of 450m.

Motion on an arc can also be considered from the view point of the kinematic equations. As pointed out earlier, displacement, velocity and acceleration are all vector quantities, i.e. they have magnitude (the speed is the magnitude of the velocity vector) and direction. This means that if one drives in a circle at constant speed one still experiences an acceleration that changes the direction. We can define a couple of parameters for objects moving on circular paths and see how they relate to the kinematic equations.

1. Tangential speed: The tangent to a circle or arc is a line that intersects the arc at exactly one point. If you were driving in a circle and instantaneously moved the steering wheel back to straight, the line you would follow would be the tangent to the circle at the point where you moved the wheel. The tangential speed then is the instantaneous magnitude of the velocity vector as one moves around the circle.

2. Tangential acceleration: The tangential acceleration is the component of acceleration that would change the tangential speed and this can be treated as a linear acceleration if one imagines that the circular path is unrolled and made linear.

3. Centripetal acceleration: Centripetal acceleration corresponds to the constant change in the direction of the velocity vector necessary to maintain a circular path. Always acting toward the center of the circle, centripetal acceleration has a magnitude proportional to the tangential speed squared divided by the radius of the path.



For the forces and equations describing uniform circular motion see section I.10.

Skill 5 Reference frames and relative motion (relative velocity, Galilean relativity)

When we analyze a situation using the laws of physics, we must first consider the perspective from which it is viewed. This is known as the frame of reference. The principles which describe the relationships between different frames of reference are known as relativities. The type of relativity discussed below is known as Galilean or Newtonian relativity and is valid for physical situations in which velocities are relatively low. When velocities approach the speed of light, we must use Einstein's special relativity (see section V.10).

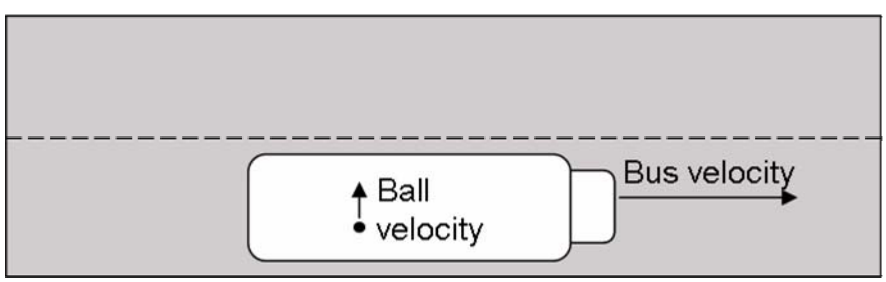
There are two general types of reference frames: inertial and non-inertial

Inertial: These frames translate at a constant vector velocity, meaning the velocity does not change direction or magnitude (i.e., travel in a straight line without acceleration).

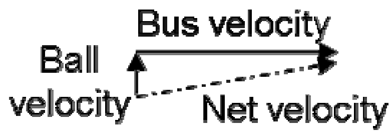
Non-inertial: These frames include all other situations in which there is non-constant velocity, such as acceleration or rotation. Galilean relativity does not apply to non-inertial frames, as explained below.

Galilean relativity states that the laws of physics are the same in all inertial frames. That is, these same laws would apply to an experiment performed on the surface of the Earth and an experiment performed in a reference frame moving at constant velocity with respect to the earth. For instance, two baseball players can have the same game of catch either standing on the ground or in a moving bus (so long as the bus's motion has constant direction and magnitude).

It is true, however, that phenomenon will have different appearance depending on our frame of reference. Relative velocity is a useful concept to help us analyze such cases. We can understand relative velocity by again considering the game of catch being played on a bus:



Inside the frame of reference of the bus, the ball travels at the velocity with which it was thrown and straight across the bus (shown by the ball velocity vector above). However, if we use stationary earth as our frame of reference, then the ball is not only moving across the bus, but down the road at the velocity with which the bus is driven. To determine the ball's velocity relative to the earth, then, we must add the ball's velocity relative to the bus and the bus's velocity relative to the earth. This can be performed with simple vector addition.



Skill 6 Force and Newton's laws of motion (Newton's first law, inertia, inertial reference frames, Newton's second law, force and acceleration, addition of forces, balanced versus unbalanced forces, Newton's third law, action-reaction forces, weight and mass)

Newton's first law of motion: "An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force". This tendency of an object to continue in its state of rest or motion is known as **inertia**. Note that, at any point in time, most objects have multiple forces acting on them. If the vector addition of all the forces on an object results in a zero net force, then the forces on the object are said to be **balanced**. If the net force on an object is non-zero, an **unbalanced** force is acting on the object.

Prior to Newton's formulation of this law, being at rest was considered the natural state of all objects because at the earth's surface we have the force of gravity working at all times which causes nearly any object put into motion to eventually come to rest. Newton's brilliant leap was to recognize that an unbalanced force changes the motion of a body, whether that body begins at rest or at some non-zero speed.

We experience the consequences of this law everyday. For instance, the first law is why seat belts are necessary to prevent injuries. When a car stops suddenly, say by hitting a road barrier, the driver continues on forward due to inertia until acted upon by a force. The seat belt provides that force and distributes the load across the whole body rather than allowing the driver to fly forward and experience the force against the steering wheel.

Newton's second law of motion: "The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object".

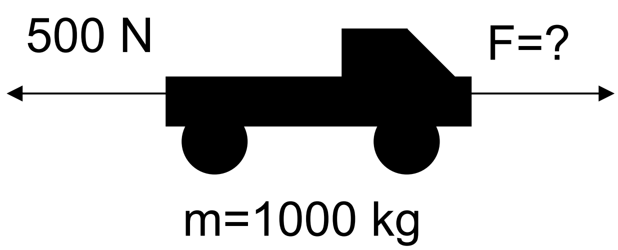
In the equation form, it is stated as ***F= ma***, force equals mass times acceleration. It is important, again, to remember that this is the net force and that forces are vector quantities. Thus if an object is acted upon by 12 forces that sum to zero, there is no acceleration. Also, this law embodies the idea of inertia as a consequence of mass. For a given force, the resulting acceleration is proportionally smaller for a more massive object because the larger object has more inertia.

The first two laws are generally applied together via the equation **F=ma.** The first law is largely the conceptual foundation for the more specific and quantitative second law. Newton's first law and second law are valid only in **inertial reference frames** (described in previous section).

The **weight** of an object is the result of the gravitational force of the earth acting on its mass. The acceleration due to Earth's gravity on an object is 9.81 m/s^2. Since force equals mass * acceleration, the magnitude of the gravitational force created by the earth on an object is

$$F_{Gravity} = m_{object} \cdot 9.81 \,{}^{m}\!/_{s^2}$$

Example: For the arrangement shown, find the force necessary to overcome the 500 N force pushing to the left and move the truck to the right with an acceleration of 5 m/s^2.



Solution: Since we know the acceleration and mass, we can calculate the net force necessary to move the truck with this acceleration. Assuming that to the right is the positive direction we sum the forces and get
F-500N = 1000kg x 5 m/s^2. Solving for F, we get 5500 N.

Newton's third law of motion: **"**For every action, there is an equal and opposite reaction". This statement means that, in every interaction, there is a pair of forces acting on the two interacting objects. The size of the force on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object.

1. The propulsion/movement of fish through water: A fish uses its fins to push water backwards. The water pushes back on the fish. Because the force on the fish is unbalanced the fish moves forward.