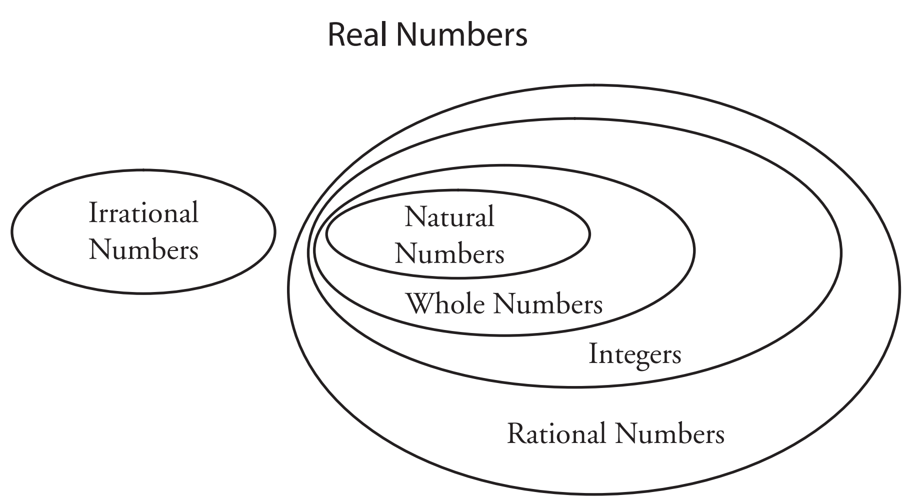
COMPETENCY 1
ALGEBRA AND NUMBER THEORY

SKILL 1.1 Demonstrate an understanding of the structure of the natural, integer, rational, real, and complex number systems and the ability to perform basic operations (+, −, ×, and ÷) on numbers in these systems

Underlying many of the more involved fields of mathematics is an understanding of basic algebra and number theory. A foundation in these concepts allows for expansion of knowledge into trigonometry, calculus, and other areas. The following discussion covers the essential properties of standard sets of numbers (such as real and complex numbers).

Real Numbers

The following chart shows the relationships among the subsets of the real numbers.



REAL NUMBERS are denoted by $\mathbb{R}$ and are numbers that can be shown by an infinite decimal representation such as 3.286275347 . . . . Real numbers include rational numbers, such as 242 and –23/129, and irrational numbers, such as $\sqrt{2}$ and π, and can be represented as points along an infinite number line. Real numbers are also known as "the unique complete Archimedean *ordered field*." Real numbers are to be distinguished from imaginary numbers.

REAL NUMBERS: numbers that can be represented by an infinite decimal representation

Real numbers are classified as follows:

CLASSIFICATIONS OF REAL NUMBERS	
Natural Numbers Denoted by $\mathbb{N}$	The counting numbers. 1, 2, 3, . . .
Whole Numbers	The counting numbers along with zero. 0, 1, 2, 3, . . .
Integers, Denoted by $\mathbb{Z}$	The counting numbers, their negatives, and zero. . . . , –2, –1, 0, 1, 2, . . .
Rationals, Denoted by $\mathbb{Q}$	All of the fractions that can be formed using whole numbers. Zero cannot be the denominator. In decimal form, these numbers will be either terminating or repeating decimals. Simplify square roots to determine if the number can be written as a fraction.
Irrationals	Real numbers that cannot be written as a fraction. The decimal forms of these numbers neither terminate nor repeat. Examples include π, e and $\sqrt{2}$.

Operations on Whole Numbers

Mathematical operations include addition, subtraction, multiplication, and division. Addition can be indicated by these expressions: *sum*, *greater than*, *and*, *more than*, *increased by*, *added to*. Subtraction can be expressed by the phrases *difference*, *fewer than*, *minus*, *less than*, *and decreased by*. Multiplication is shown by *product*, *times*, *multiplied by*, and *twice*. Division is indicated by *quotient*, *divided by*, and *ratio*.

Addition and subtraction

There are two main procedures used in addition and subtraction: adding or subtracting single digits and “carrying” or “borrowing.”

Example: Find the sum of 346 + 225 using place value.

PLACE VALUE 100	10	1
3 2	4 2	6 5
5	6	11
5	7	1

Standard algorithm for addition check:

$$\begin{array}{r} 3\ {}^{1}4\ 6 \\ +\,2\ \ 2\ 5 \\ \hline 5\ \ 7\ 1 \end{array}$$

Example: Find the difference 234 − 46 using place value.

PLACE VALUE 100	10	1
1	13 4	4 6

PLACE VALUE 100	10	1
1	12 4	14 6
1	**8**	**8**

Standard algorithm for subtraction check:

$$\begin{array}{r} 2\ 3\ 4 \\ -\ \ \ 4\ 6 \\ \hline 1\ 8\ 8 \end{array}$$

Multiplication and division

Multiplication is one of the four basic number operations. In simple terms, multiplication is the addition of a number to itself a certain number of times. For example, 4 multiplied by 3 is the equal to $4 + 4 + 4$ or $3 + 3 + 3 + 3$. Another way of conceptualizing multiplication is to think in terms of groups. For example, if we have 4 groups of 3 students, the total number of students is 4 multiplied by 3. We call the solution to a multiplication problem the **PRODUCT**.

PRODUCT: a solution to a multiplication problem

Example: A student buys 4 boxes of crayons. Each box contains 16 crayons. How many total crayons does the student have?

The total number of crayons is 16×4.

$$\begin{array}{r} 16 \\ \times \quad 4 \\ \hline 64 \end{array}$$

Total number of crayons equals 64 crayons.

Division, the inverse of multiplication, is another of the four basic number operations. When we divide one number by another, we determine how many times we can multiply the divisor (number divided by) before we exceed the number we are dividing (dividend). For example, 8 divided by 2 equals 4 because we can multiply 2 four times to reach 8 ($2 \times 4 = 8$ or $2 + 2 + 2 + 2 = 8$). Using the grouping conceptualization we used with multiplication, we can divide 8 into 4 groups of 2 or 2 groups of 4. We call the answer to a division problem the **QUOTIENT**.

QUOTIENT: the answer to a division problem

If the divisor does not divide evenly into the dividend, we express the leftover amount either as a remainder or as a fraction with the divisor as the denominator. For example, 9 divided by 2 equals 4 with a remainder of 1 or $4\frac{1}{2}$.

Example: Each box of apples contains 24 apples. How many boxes must a grocer purchase to supply a group of 252 people with one apple each?

The grocer needs 252 apples. Because he must buy apples in groups of 24, we divide 252 by 24 to determine how many boxes he needs to buy.

$$\begin{array}{r} 10 \\ 24\overline{)252} \\ -\,24 \\ \hline 12 \\ -\,0 \\ \hline 12 \end{array}$$

$\rightarrow$ The quotient is 10 with a remainder of 12.

Thus, the grocer needs 10 full boxes plus 12 more apples. Therefore, the minimum number of boxes the grocer must purchase is 11 boxes.

Adding and Subtracting Decimals

When adding and subtracting decimals, we align the numbers by place value as we do with whole numbers. After adding or subtracting each column, we bring the decimal down, placing it in the same location as in the numbers added or subtracted.

Example: Find the sum of 152.3 and 36.342.

$$\begin{array}{r} 152.300 \\ \underline{+\ 36.342} \\ 188.642 \end{array}$$

Note that we placed two zeroes after the final place value in 152.3 to clarify the column addition.

Example: Find the difference of 152.3 and 36.342.

2 9 10	(4)11(12)
152.~~300~~	1~~52.300~~
− 36.342	− 36.342
58	115.958

Note how we borrowed to subtract from the zeros in the hundredths and thousandths place of 152.300.

Operations with Signed Numbers

When adding numbers with the same sign, the result will also have the same sign. When adding numbers that have different signs, subtract the smaller number from the larger number (ignoring the sign) and then use the sign of the larger number. When subtracting a negative number, change the sign of the number to a positive sign and then add it (i.e., replace the two negative signs by a positive sign).

Examples:

$(3) + (4) = 7$

$(-8) + (-4) = -12$

$(6) - (5) = 1$

$(3) - (6) = -3$

$(-4) - (2) = -6$

$(-6) - (-10) = 4$

When we multiply two numbers with the same sign, the result is positive. If the two numbers have different signs, the result is negative. The same rule follows for division.

Examples:

$(5)(5) = 25$
$(5)(-6) = -30$
$(-19)(-2) = 38$
$16 \div 4 = 4$
$(-34) \div 2 = -17$
$(-18) \div (-2) = 9$
$27 \div (-3) = -9$

Order of Operations

The **Order of Operations** is to be followed when evaluating expressions with multiple operations. Remember the mnemonic PEMDAS (Please Excuse My Dear Aunt Sally) to follow these steps in order:

1. Simplify inside grouping characters such as parentheses, brackets, radicals, fraction bars, etc.
2. Multiply out expressions with exponents.
3. Do multiplication or division from left to right.

   Note: Multiplication and division are equivalent even though multiplication is mentioned before division in the mnemonic PEMDAS.
4. Do addition or subtraction from left to right.

   Note: Addition and subtraction are equivalent even though addition is mentioned before “subtraction in the mnemonic PEMDAS.

Example:

Evaluate: $\frac{12(9-7)+4\times 5}{3^4+2^3}$

$\frac{12(9-7)+4\times 5}{3^4+2^3}$

$= \frac{12(2)+4\times 5}{3^4+2^3}$ Simplify within parentheses.

$= \frac{12(2)+4\times 5}{81+8}$ Multiply out exponent expressions.

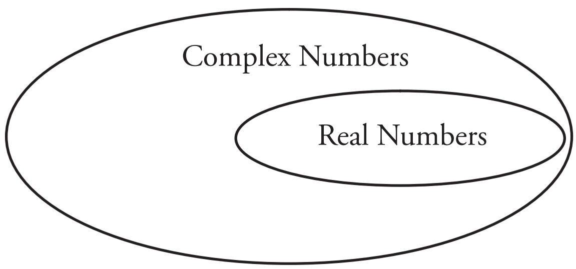
$= \frac{24+20}{81+8}$ Do multiplication and division.

$= \frac{44}{89}$ Do addition and subtraction.

Complex Numbers

The set of complex numbers is denoted by $\mathbb{C}$. The set $\mathbb{C}$ is defined as $\{a + bi : a, b \in \mathbb{R}\}$ ($\in$ means "element of"). In other words, complex numbers are an extension of real numbers made by attaching an imaginary number i, which satisfies the equality $i^2 = -1$. **COMPLEX NUMBERS** are of the form $\boldsymbol{a + bi}$, where a and b are real numbers and $i = \sqrt{-1}$. Thus, a is the real part of the number and b is the imaginary part of the number. When i appears in a fraction, the fraction is usually simplified so that i is not in the denominator. The set of complex numbers includes the set of real numbers, where any real number n can be written in its equivalent complex form as $n + 0i$. In other words, it can be said that $\mathbb{R} \subseteq \mathbb{C}$ (or $\mathbb{R}$ is a subset of $\mathbb{C}$).

COMPLEX NUMBERS: numbers of the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$



The number $3i$ has a real part 0 and imaginary part 3; the number 4 has a real part 4 and an imaginary part 0. As another way of writing complex numbers, we can express them as ordered pairs:

Complex number	Ordered pair
$3 + 2i$	(3, 2)
$\sqrt{3} + \sqrt{3}i$	$(\sqrt{3}, \sqrt{3})$
$7i$	(0, 7)
$\frac{6 + 2i}{7}$	$\left(\frac{6}{7}, \frac{2}{7}\right)$

The basic operations for complex numbers can be summarized as follows, where $z_1 = a_1 + b_1i$ and $z_2 = a_2 + b_2i$. Note that the operations are performed in the standard manner, where i is treated as a standard radical value. The result of each operation is written in the standard form for complex numbers. Also note that the **COMPLEX CONJUGATE** of a complex number $z = a + bi$ is denoted as $z^* = a - bi$.

COMPLEX CONJUGATE: for a complex number $z = a + bi$, this is denoted as $z^* = a - bi$

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$$
$$z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)i$$
$$z_1z_2 = (a_1a_2 - b_1b_2) + (a_1b_2 - a_2b_1)i$$
$$\frac{z_1}{z_2} = \frac{z_1}{z_2}\frac{z_2^*}{z_2^*} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + \frac{a_2b_1 + a_1b_2}{a_2^2 + b_2^2}i$$

SKILL 1.2 **Compare and contrast properties** *(e.g., closure, commutativity, associativity, distributivity)* **of number systems under various operations**

Fields and Rings

FIELD: any set that includes at least two nonzero elements that satisfies the field axioms for addition and multiplication

Any set that includes at least two nonzero elements that satisfies the field axioms for addition and multiplication is a **FIELD**. The real numbers, $\mathbb{R}$, as well as the complex numbers, $\mathbb{C}$, are each a field, with the real numbers being a subset of the complex numbers. The field axioms are summarized below.

FIELD AXIOMS	
ADDITION	
Commutativity	$a + b = b + a$
Associativity	$a + (b + c) = (a + b) + c$
Identity	$a + 0 = a$
Inverse	$a + (-a) = 0$
MULTIPLICATION	
Commutativity	$ab = ba$
Associativity	$a(bc) = (ab)c$
Identity	$a \times 1 = a$
Inverse	$a \times \frac{1}{a} = 1 \quad (a \neq 0)$
ADDITION AND MULTIPLICATION	
Distributivity	$a(b + c) = (b + c)a = ab + ac$

Note that both the real numbers and the complex numbers satisfy the axioms summarized above.

RING: an integral domain with two binary operations (addition and multiplication) where, for every nonzero element a and b in the domain, the product *ab* is nonzero

Note: *Multiplication is implied when there is no symbol between two variables. Thus,* $a \times b$ *can be written* ab*. Multiplication can also be indicated by a raised dot* ($\cdot$)*.*

A **RING** is an integral domain with two binary operations (addition and multiplication) where, for every nonzero element a and b in the domain, the product ab is nonzero. A field is a ring in which multiplication is commutative, or $a \times b = b \times a$, and all nonzero elements have a multiplicative inverse. The set $\mathbb{Z}$ (integers) is a ring that is not a field in that it does not have the multiplicative inverse; therefore, integers are not a field. A polynomial ring is also not a field, as it also has no multiplicative inverse. Furthermore, matrix rings do not constitute fields because matrix multiplication is not generally commutative.

Real numbers are an ordered field and can be ordered

As such, an ordered field F must contain a subset P (such as the positive numbers) such that if a and b are elements of P, then both $a + b$ and ab are also elements of P. (In other words, the set P is closed under addition and multiplication.) Furthermore, it must be the case that for any element c contained in F, exactly one of the following conditions is true: c is an element of P, $-c$ is an element of P, or $c = 0$.

The rational numbers also constitute an ordered field

The set P can be defined as the positive rational numbers. For each a and b that are elements of the set $\mathbb{Q}$ (the rational numbers), $a + b$ is also an element of P, as is ab. (The sum $a + b$ and the product ab are both rational if a and b are rational.) Since P is closed under addition and multiplication, $\mathbb{Q}$ constitutes an ordered field.

Complex numbers, unlike real numbers, cannot be ordered

Consider the number $i = \sqrt{-1}$ contained in the set $\mathbb{C}$ of complex numbers. Assume that $\mathbb{C}$ has a subset P (positive numbers) that is closed under both addition and multiplication. Assume that $i > 0$. A difficulty arises in that $i^2 = -1 < 0$, so i cannot be included in the set P. Likewise, assume $i < 0$. The problem once again arises that $i^4 = 1 > 0$, so i cannot be included in P. It is clearly the case that $i \neq 0$, so there is no place for i in an ordered field. Thus, the complex numbers cannot be ordered.

Example: Prove that for every integer y, if y is an even number, then y^2 is even.

The definition of *even* implies that for each integer y there is at least one integer x such that $y = 2x$.

$$y = 2x$$
$$y^2 = 4x^2$$

Since $4x^2$ is always evenly divisible by two ($2x^2$ is an integer), y^2 is even for all values of y.

Example: If a, b, and c are positive real numbers, prove that $c(a + b) = (b + a)c$.

Use the properties of the set of real numbers.

$c(a + b)$	$= c(b + a)$	Additive commutativity
	$= cb + ca$	Distributivity
	$= bc + ac$	Multiplicative commutativity
	$= (b + a)c$	Distributivity

Example: Given real numbers a, b, c, and d, where ad = –bc, prove that (a + bi)(c + di) is real.

Expand the product of the complex numbers.

$$(a + bi)\,(c + di) = ac + bci + adi + bdi^2$$

Use the definition of i^2.

$$(a + bi)\,(c + di) = ac - bd + bci + adi$$

Apply the fact that ad = –bc.

$$(a + bi)\,(c + di) = ac - bd + bci - bci = ac - bd$$

Since a, b, c and d are all real, $ac - bd$ must also be real.

Closure

CLOSURE: a set is closed under an operation if the operation performed on any given elements of the set always yields a result that is likewise an element of the set

Another useful property that can describe arbitrary sets of numbers (including fields and rings) is **CLOSURE**. A set is closed under an operation if the operation performed on any given elements of the set always yields a result that is likewise an element of the set. For instance, the set of real numbers is closed under multiplication, because for any two real numbers a and b, the product ab is also a real number.

Example: Determine if the set of integers is closed under division.

For the set of integers to be closed under division, it must be the case that $\frac{a}{b}$ is an integer for any integers a and b. Consider $a = 2$ and $b = 3$.

$$\frac{a}{b} = \frac{2}{3}$$

This result is not an integer. Therefore, the set of integers is not closed under division.

SKILL 1.3 Demonstrate an understanding of the properties of counting numbers *(e.g., prime, composite, prime factorization, even, odd, factors, multiples)*

Natural (Counting) Numbers

NATURAL NUMBERS: the numbers 1, 2, 3, 4, . . . ; they are sometimes called the counting numbers

The set of **NATURAL NUMBERS**, ℕ, includes 1, 2, 3, 4, ... (For some definitions, ℕ includes zero.) The natural numbers are sometimes called the **counting numbers** (especially if the definition of ℕ excludes zero). The set ℕ constitutes neither a ring nor a field, because there is no additive inverse (since there are no negative numbers).

The set $\mathbb{N}$ obeys the properties of associativity, commutativity, distributivity and identity for multiplication and addition (assuming, for the case of addition, that zero is included in some sense in the natural numbers). The set of natural numbers does *not* obey additive or multiplicative inverses, however, as there are no noninteger fractions or negative numbers.

Natural numbers can be either *even* or *odd*. Even numbers are evenly divisible by two; odd numbers are not evenly divisible by two (alternatively, they leave a remainder of one when divided by two). Any natural number n that is divisible by at least one number that is not equal to 1 or n is called a **COMPOSITE NUMBER**. A natural number n that is only divisible by 1 or n is called a **PRIME NUMBER**.

COMPOSITE NUMBER: any natural number n that is divisible by at least one number that is not equal to 1 or n

PRIME NUMBER: a natural number n that is only divisible by 1 or n

Divisibility tests

1. A number is *divisible by 2* if that number is an even number (i.e., the last digit is 0, 2, 4, 6 or 8).

   Consider a number $abcd$ defined by the digits a, b, c and d (for instance, 1,234). Rewrite the number as follows.

   $$10abc + d = abcd$$

   Note that $10abc$ is divisible by 2. Thus, the number $abcd$ is only divisible by 2 if d is divisible by two; in other words, $abcd$ is divisible by two only if it is an even number. For example, the last digit of 1,354 is 4, so it is divisible by 2. On the other hand, the last digit of 240,685 is 5, so it is not divisible by 2.

2. A number is *divisible by 3* if the sum of its digits is evenly divisible by 3.

   Consider a number $abcd$ defined by the digits a, b, c and d. The number can be written as

   $$abcd = 1000a + 100b + 10c + d$$

   The number can also be rewritten as

   $$abcd = (999 + 1)a + (99 + 1)b + (9 + 1)c + d$$
   $$abcd = 999a + 99b + 9c + (a + b + c + d)$$

   Note that the first three terms in the above expression are all divisible by 3. Thus, the number is evenly divisible by 3 only if $a + b + c + d$ is divisible by 3. The same logic applies regardless of the size of the number. This proves the rules for divisibility by 3.

   The sum of the digits of 964 is $9 + 6 + 4 = 19$. Since 19 is not divisible by 3, neither is 964. The digits of 86,514 is $8 + 6 + 5 + 1 + 4 = 24$. Since 24 is divisible by 3, 86,514 is also divisible by 3.

3. A number is *divisible by 4* if the number in its last two digits is evenly divisible by 4.

Let a number *abcd* be defined by the digits *a*, *b*, *c* and *d*.

$ab(100) + cd = abcd$

Since 100 is divisible by 4, 100*ab* is also divisible by 4. Thus, *abcd* is divisible by 4 only if *cd* is divisible by 4.

$25ab + \frac{cd}{4} = \frac{abcd}{4}$

The number 113,336 ends with the number 36 for the last two digits. Since 36 is divisible by 4, 113,336 is also divisible by 4. The number 135,627 ends with the number 27 for the last two digits. Since 27 is not evenly divisible by 4, 135,627 is also not divisible by 4.

4. A number is *divisible by 5* if the number ends in either a 5 or a 0.

Use the same number abcd.

$100ab + cd = abcd$

The first term is evenly divisible by 5, but the second term is only evenly divisible by 5 if it is 0, 5, 10, 15, . . . , 95. In other words, *abcd* is divisible by 5 only if it ends in a 0 or a 5. For instance, 225 ends with a 5, so it is divisible by 5. The number 470 is also divisible by 5 because its last digit is a 0. The number 2,358 is not divisible by 5 because its last digit is an 8.

5. A number is *divisible by 6* if the number is divisible by both 2 and 3. Thus any even number that is divisible by 3 is also divisible by 6. For instance, 4,950 is an even number and its digits add up to 18 (4 + 9 + 5 + 0 = 18). Since it is even and the sum of its digits is divisible by 3, the number 4,950 is divisible by 3 and by 6 as well. On the other hand, 326 is an even number, but its digits add up to 11. Since 11 is not divisible by 3, 326 is not divisible by 3 or by 6.

6. A number is *divisible by 8* if the number in its last three digits is evenly divisible by 8.

The logic for the proof of this case follows that of numbers divisible by 2 and 4. The number 113,336 ends with the 3-digit number 336 in the last three columns. Since 336 is divisible by 8, then 113,336 is also divisible by 8. The number 465,627 ends with the number 627 in the last three columns. Since 627 is not evenly divisible by 8, then 465,627 is also not divisible by 8.

7. A number is *divisible by 9* if the sum of its digits is evenly divisible by 9.

The logic for the proof of this case follows that for the case of numbers that are divisible by 3 and 6. The sum of the digits of 874, for example, is 8 +

7 + 4 = 19. Since 19 is not divisible by 9, neither is 874. The sum of the digits of 116,514 is 1 + 1 + 6 + 5 + 1 + 4 = 18. Since 18 is divisible by 9, 116,514 is also divisible by 9.

The Fundamental Theorem of Arithmetic

Every integer greater than 1 can be written uniquely in the form

$$p_1^{e1}\, p_2^{e2} \cdots p_k^{ek}.$$

The p_i are distinct prime numbers and the e_i are positive integers.

Greatest common factor

GCF is the abbreviation for the **GREATEST COMMON FACTOR**. The GCF is the largest number that is a factor of all the numbers given in a problem. The GCF can be no larger than the smallest number given in the problem. If no other number is a common factor, then the GCF will be the number 1. To find the GCF, list all possible factors of the smallest number given (include the number itself). Starting with the largest factor (which is the number itself), determine if it is also a factor of all the other given numbers. If so, that is the GCF. If that factor does not work, try the same method on the next smaller factor. Continue until a common factor is found. This is the GCF. Note: There can be other common factors besides the GCF.

GREATEST COMMON FACTOR: the largest number that is a factor of all the numbers given in a problem

The GCF can be no larger than the smallest number given in the problem.

Example: Find the GCF of 12, 20, and 36.

The smallest number in the problem is 12. The factors of 12 are 1, 2, 3, 4, 6 and 12. 12 is the largest factor, but it does not divide evenly into 20. Neither does 6, but 4 will divide into both 20 and 36 evenly. Therefore, 4 is the GCF.

Example: Find the GCF of 14 and 15.

Factors of 14 are 1, 2, 7 and 14. 14 is the largest factor, but it does not divide evenly into 15. Neither does 7 or 2. Therefore, the only factor common to both 14 and 15 is the number 1, which is the GCF.

The **Euclidean Algorithm** is a formal method for determining the **greatest common divisor** (GCD, another name for GCF) of two positive integers. The algorithm can be formulated in a recursive manner that simply involves repetition of a few steps until a terminating point is reached. The algorithm can be summarized as follows, where a and b are the two integers for which determination of the GCD is to be undertaken. (Assign a and b such that $a > b$.)

1. If $b = 0$, a is the GCD
2. Calculate $c = a \bmod b$

3. If $c = 0$, b is the GCD

4. Go back to step 2, replacing a with b and b with c

Note that the mod operator in this case is simply a remainder operator. Thus, a mod b is the remainder of division of a by b.

Example: Find the GCD of 299 and 351.

To find the GCD, first let $a = 351$ and $b = 299$. Begin the algorithm as follows.

Step 1: $b \neq 0$
Step 2: $c = 351 \bmod 299 = 52$
Step 3: $c \neq 0$

Perform the next iteration, starting with step 2.

Step 2: $c = 299 \bmod 52 = 39$
Step 3: $c \neq 0$

Continue to iterate recursively until a solution is found.

Step 2: $c = 52 \bmod 39 = 13$
Step 3: $c \neq 0$

Step 2: $c = 39 \bmod 13 = 0$
Step 3: $c = 0$: GCD $= 13$

Thus, the GCD of 299 and 351 is thus 13.

Least common multiple

LEAST COMMON MULTIPLE: of a group of numbers is the smallest number that all of the given numbers will divide into

LCM is the abbreviation for **LEAST COMMON MULTIPLE**. The least common multiple of a group of numbers is the smallest number that all of the given numbers will divide into. The least common multiple will always be the largest of the given set of numbers, or a multiple of the largest number.

The least common multiple will always be the largest of the given set of numbers, or a multiple of the largest number.

Example: Find the LCM of 20, 30 and 40.

The largest number given is 40, but 30 will not divide evenly into 40. The next multiple of 40 is 80 (2×40), but 30 will not divide evenly into 80 either. The next multiple of 40 is 120. 120 is divisible by both 20 and 30, so 120 is the LCM (least common multiple).

Example: Find the LCM of 96, 16 and 24.

The largest number is 96. 96 is divisible by both 16 and 24, so 96 is the LCM.