

COMPETENCY 1.0 KNOWLEDGE OF NUMBER SENSE, CONCEPTS, AND OPERATIONS

Skill 1.1 Compare the relative value of real numbers (e.g., integers, fractions, decimals, percents, irrational numbers, and numbers expressed in exponential or scientific notation).

We can express **rational numbers** as the ratio of two integers $\frac{a}{b}$, where $b \neq 0$. For

example, $\frac{2}{3}$, $-\frac{4}{5}$, $5 = \frac{5}{1}$.

The rational numbers include integers, fractions, mixed numbers, and terminating and repeating decimals. We can express every rational number as a repeating or terminating decimal and represent it on a number line.

Integers are positive and negative whole numbers and zero.

...-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, ...

Whole numbers are natural numbers and zero.

0, 1, 2, 3, 4, 5, 6, ...

Natural numbers are the counting numbers.

1, 2, 3, 4, 5, 6, ...

Irrational numbers are real numbers that we cannot write as the ratio of two integers. These are infinite non-repeating decimals.

Examples: $\sqrt{5} = 2.2360\dots$, $\pi = 3.1415927\dots$

A **fraction** is an expression of numbers in the form of x/y , where x is the numerator and y is the denominator, which cannot be zero.

Example: $\frac{3}{7}$ 3 is the numerator, 7 is the denominator

If the fraction has common factors for the numerator and denominator, divide both by the common factor to reduce the fraction to its lowest form.

Example:

$$\frac{13}{39} = \frac{1 \times 13}{3 \times 13} = \frac{1}{3}$$

Divide by the common factor 13

A **mixed** number has an integer part and a fractional part.

Example: $2\frac{1}{4}$, $-5\frac{1}{6}$, $7\frac{1}{3}$

Percent = per 100 (written with the symbol %). Thus $10\% = \frac{10}{100} = \frac{1}{10}$.

Decimals = deci = part of ten. To find the decimal equivalent of a fraction, use the denominator to divide the numerator as shown in the following example.

Example: Find the decimal equivalent of $\frac{7}{10}$.

Since 10 cannot divide into 7 evenly

$$\frac{7}{10} = 0.7$$

The **exponent form** is a shortcut method to write repeated multiplication. Basic form: b^n , where b is the base and n is the exponent. b and n are both real numbers. b^n indicates that we multiply the base, b , by itself n times.

Examples: $3^4 = 3 \times 3 \times 3 \times 3 = 81$

$$2^3 = 2 \times 2 \times 2 = 8$$
$$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$$
$$-2^4 = -(2 \times 2 \times 2 \times 2) = -16$$

Key exponent rules:

For ' a ' (nonzero), and ' m ' and ' n ' (real numbers):

- 1) $a^m \cdot a^n = a^{(m+n)}$ Product rule
- 2) $\frac{a^m}{a^n} = a^{(m-n)}$ Quotient rule
- 3) $\frac{a^{-m}}{a^{-n}} = \frac{a^n}{a^m}$

When we raise 10 to any power, the exponent tells us the number of zeroes in the product.

Example: $10^7 = 10,000,000$

Caution: Unless the negative sign is inside the parentheses and the exponent is outside the parentheses, the exponent does not affect the sign.

$(-2)^4$ implies that we multiply -2 by itself 4 times.

-2^4 implies that we multiply 2 by itself 4 times, then negate the answer.

Scientific notation is a more convenient method for writing very large and very small numbers. It employs two factors. The first factor is a number between 1 and 10. The second factor is a power of 10. This notation is a “shorthand” for expressing large numbers (like the weight of 100 elephants) or small numbers (like the weight of an atom in pounds).

Recall that:

$10^n = (10)^n$ Ten multiplied by itself n times.

$10^0 = 1$ Any nonzero number raised to power of zero is 1.

$10^1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1000$ (kilo)

$10^{-1} = 1/10$ (deci)

$10^{-2} = 1/100$ (centi)

$10^{-3} = 1/1000$ (milli)

$10^{-6} = 1/1,000,000$ (micro)

Example: Write 46,368,000 in scientific notation.

- 1) Introduce a decimal point and decimal places.
 $46,368,000 = 46,368,000.0000$
- 2) Make a mark between the two digits that give a number between -9.9 and 9.9.
 $4 \wedge 6,368,000.0000$
- 3) Count the number of digit places between the decimal point and the \wedge mark. This number is the ‘ n ’-the power of ten.

So, $46,368,000 = 4.6368 \times 10^7$

Example: Write 0.00397 in scientific notation.

- 1) Decimal place is already in place.
- 2) Make a mark between 3 and 9 to form a number between -9.9 and 9.9.
- 3) Move decimal place to the mark (3 hops).

$$0.003 \overset{\wedge}{9}7$$

Motion is to the right, so n of 10^n is negative.

$$\text{Therefore, } 0.00397 = 3.97 \times 10^{-3}.$$

Skill 1.2 Solve real-world problems involving addition, subtraction, multiplication, and division of rational numbers (e.g., whole numbers, integers, decimals, percents, and fractions including mixed numbers).

Properties are rules that apply for addition, subtraction, multiplication, or division of real numbers. These properties are:

Commutative: You can change the order of the terms or factors as follows.

$$\text{For addition: } a + b = b + a$$

$$\text{For multiplication: } ab = ba$$

Associative: You can regroup the terms as you like.

$$\text{For addition: } a + (b + c) = (a + b) + c$$

$$\text{For multiplication: } a(bc) = (ab)c$$

This rule does not apply for division and subtraction.

$$\begin{aligned} \text{Example: } (-2 + 7) + 5 &= -2 + (7 + 5) \\ 5 + 5 &= -2 + 12 = 10 \end{aligned}$$

$$\begin{aligned} \text{Example: } (3 \times -7) \times 5 &= 3 \times (-7 \times 5) \\ -21 \times 5 &= 3 \times -35 = -105 \end{aligned}$$

Identity:

A number that when added to a term results in that same number (additive identity); a number that when multiplied by a term results in that same number (multiplicative identity).

For addition: $a + 0 = a$ (zero is additive identity)

For multiplication: $a \times 1 = a$ (one is multiplicative)

Example: $17 + 0 = 17$

Example: $^{-}34 \times 1 = ^{-}34$

The product of any number and one is that number.

Distributive:

This technique allows us to operate on terms within a parentheses without first performing operations within the parentheses. This is especially helpful when we cannot combine terms within the parentheses.

$$a(b + c) = ab + ac$$

Example: $6 \times (^{-}4 + 9) = (6 \times ^{-}4) + (6 \times 9)$
 $6 \times 5 = ^{-}24 + 54 = 30$

To multiply a sum by a number, multiply each addend by the number, then add the products.

Addition of whole numbers

Example: At the end of a day of shopping, a shopper had \$24 remaining in his wallet. He spent \$45 on various goods. How much money did the shopper have at the beginning of the day?

The total amount of money the shopper started with is the sum of the amount spent and the amount remaining at the end of the day.

$$\begin{array}{r} 24 \\ + 45 \\ \hline 69 \end{array} \longrightarrow \text{The original total was \$69.}$$

Example: The winner of a race took 1 hr. 58 min. 12 sec. on the first half of the race and 2 hr. 9 min. 57 sec. on the second half of the race. What was the winner's total time?

$$\begin{array}{r} 1 \text{ hr. } 58 \text{ min. } 12 \text{ sec.} \\ + 2 \text{ hr. } 9 \text{ min. } 57 \text{ sec.} \\ \hline 3 \text{ hr. } 67 \text{ min. } 69 \text{ sec.} \\ + 1 \text{ min. } - 60 \text{ sec.} \\ \hline 3 \text{ hr. } 68 \text{ min. } 9 \text{ sec.} \\ + 1 \text{ hr. } - 60 \text{ min.} \\ \hline 4 \text{ hr. } 8 \text{ min. } 9 \text{ sec.} \end{array} \begin{array}{l} \text{Add these numbers} \\ \text{Change 60 seconds to 1min.} \\ \text{Change 60 minutes to 1 hr.} \\ \leftarrow \text{Final answer} \end{array}$$

Subtraction of Whole Numbers

Example: At the end of his shift, a cashier has \$96 in the cash register. At the beginning of his shift, he had \$15. How much money did the cashier collect during his shift?

The total collected is the difference of the ending amount and the starting amount.

$$\begin{array}{r} 96 \\ - 15 \\ \hline 81 \end{array} \longrightarrow \text{The total collected was \$81.}$$

Multiplication of whole numbers

Multiplication is one of the four basic number operations. In simple terms, multiplication is the addition of a number to itself a certain number of times. For example, 4 multiplied by 3 is the equal to $4 + 4 + 4$ or $3 + 3 + 3 + 3$. Another way of conceptualizing multiplication is to think in terms of groups. For example, if we have 4 groups of 3 students, the total number of students is 4 multiplied by 3. We call the solution to a multiplication problem the product.

The basic algorithm for whole number multiplication begins with aligning the numbers by place value with the number containing more places on top.

$$\begin{array}{r} 172 \\ \times 43 \\ \hline \end{array} \longrightarrow \text{Note that we placed 122 on top because it has more places than 43 has.}$$

Next, we multiply the ones' place of the second number by each place value of the top number sequentially.

$$\begin{array}{r} (2) \\ 172 \\ \times 43 \\ \hline 516 \end{array} \longrightarrow \{3 \times 2 = 6, 3 \times 7 = 21, 3 \times 1 = 3\}$$

Note that we had to carry a 2 to the hundreds' column because $3 \times 7 = 21$. Note also that we add, not multiply, carried numbers to the product.

Next, we multiply the number in the tens' place of the second number by each place value of the top number sequentially. Because we are multiplying by a number in the tens' place, we place a zero at the end of this product.

$$\begin{array}{r} (2) \\ 172 \\ \times 43 \\ \hline 516 \\ 6880 \end{array} \longrightarrow \{4 \times 2 = 8, 4 \times 7 = 28, 4 \times 1 = 4\}$$

Finally, to determine the final product we add the two partial products.

$$\begin{array}{r} 172 \\ \times 43 \\ \hline 516 \\ + 6880 \\ \hline 7396 \end{array} \longrightarrow \text{The product of 172 and 43 is 7396.}$$

Example: A student buys 4 boxes of crayons. Each box contains 16 crayons.
How many total crayons does the student have?

The total number of crayons is 16×4 .

$$\begin{array}{r} 16 \\ \times 4 \\ \hline 64 \end{array} \longrightarrow \text{Total number of crayons equals 64.}$$

Division of whole numbers

Division, the inverse of multiplication, is another of the four basic number operations. When we divide one number by another, we determine how many times we can multiply the divisor (number divided by) before we exceed the number we are dividing (dividend). For example, 8 divided by 2 equals 4 because we can multiply 2 four times to reach 8 ($2 \times 4 = 8$ or $2 + 2 + 2 + 2 = 8$). Using the grouping conceptualization we used with multiplication, we can divide 8 into 4 groups of 2 or 2 groups of 4. We call the answer to a division problem the quotient.

If the divisor does not divide evenly into the dividend, we express the leftover amount either as a remainder or as a fraction with the divisor as the denominator. For example, 9 divided by 2 equals 4 with a remainder of 1 or $4 \frac{1}{2}$.

The basic algorithm for division is long division. We start by representing the quotient as follows.

$$14 \overline{)293} \longrightarrow 14 \text{ is the divisor and } 293 \text{ is the dividend.}$$

This represents $293 \div 14$.

Next, we divide the divisor into the dividend starting from the left.

$$14 \overline{)293} \xrightarrow{2} 14 \text{ divides into } 29 \text{ two times with a remainder.}$$

Next, we multiply the partial quotient by the divisor, subtract this value from the first digits of the dividend, and bring down the remaining dividend digits to complete the number.

$$\begin{array}{r} 2 \\ 14 \overline{)293} \\ \underline{-28} \\ 13 \end{array} \longrightarrow 2 \times 14 = 28, 29 - 28 = 1, \text{ and bringing down the } 3 \text{ yields } 13.$$

Finally, we divide again (the divisor into the remaining value) and repeat the preceding process. The number left after the subtraction represents the remainder.

$$\begin{array}{r}
 14 \overline{)293} \\
 \underline{-28} \\
 13 \\
 \underline{-0} \\
 13
 \end{array}$$

→ The final quotient is 20 with a remainder of 13. We can also represent this quotient as $20 \frac{13}{14}$.

Example: Each box of apples contains 24 apples. How many boxes must a grocer purchase to supply a group of 252 people with one apple each?

The grocer needs 252 apples. Because he must buy apples in groups of 24, we divide 252 by 24 to determine how many boxes he needs to buy.

$$\begin{array}{r}
 10 \\
 24 \overline{)252} \\
 \underline{-24} \\
 12 \\
 \underline{-0} \\
 12
 \end{array}$$

→ The quotient is 10 with a remainder of 12.

Thus, the grocer needs 10 boxes plus 12 more apples. Therefore, the minimum number of boxes the grocer can purchase is 11.

Example: At his job, John gets paid \$20 for every hour he works. If John made \$940 in a week, how many hours did he work?

This is a division problem. To determine the number of hours John worked, we divide the total amount made (\$940) by the hourly rate of pay (\$20). Thus, the number of hours worked equals 940 divided by 20.

$$\begin{array}{r}
 47 \\
 20 \overline{)940} \\
 \underline{-80} \\
 140 \\
 \underline{-140} \\
 0
 \end{array}$$

→ 20 divides into 940, 47 times with no remainder.

John worked 47 hours.

Addition and Subtraction of Decimals

When adding and subtracting decimals, we align the numbers by place value as we do with whole numbers. After adding or subtracting each column, we bring the decimal down, placing it in the same location as in the numbers added or subtracted.

Example: Find the sum of 152.3 and 36.342.

$$\begin{array}{r} 152.300 \\ + 36.342 \\ \hline 188.642 \end{array}$$

Note that we placed two zeroes after the final place value in 152.3 to clarify the column addition.

Example: Find the difference of 152.3 and 36.342.

$$\begin{array}{r} 9 \\ 152.300 \\ - 36.342 \\ \hline 58 \end{array} \quad \longrightarrow \quad \begin{array}{r} (4)11(12) \\ 152.300 \\ - 36.342 \\ \hline 115.958 \end{array}$$

Note how we borrowed to subtract from the zeroes in the hundredths' and thousandths' place of 152.300.

Multiplication of Decimals

When multiplying decimal numbers, we multiply exactly as with whole numbers and place the decimal moving in from the left the total number of decimal places contained in the two numbers multiplied. For example, when multiplying 1.5 and 2.35, we place the decimal in the product 3 places in from the left (3.525).

Example: Find the product of 3.52 and 4.1.

$$\begin{array}{r} 3.52 \\ \times 4.1 \\ \hline 352 \\ + 14080 \\ \hline 14432 \end{array}$$

→ Note that there are 3 total decimal places in the two numbers.

→ We place the decimal 3 places in from the left.

Thus, the final product is 14.432.

